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Modular functions, Jacobi forms and mock modular forms appear naturally in various contexts in string theory and conformal field theory. In particular, characters of conformal field theories (CFTs) define (vector-valued) modular functions, while Jacobi forms and mock modular forms arise from the elliptic genus of superstring compactifications. For example, the J-function is the character of the famous Monster CFT, while the unique weak Jacobi form of weight zero and index one, φ0,1, is the elliptic genus of the superstring compactification on a K3 surface.

About 30 years ago, it was noted that the Fourier coefficients of the J-function can be interpreted in terms of representations of the Monster group, the largest simple sproadic finite group, and this gave rise to a development that is now usually referred to as `Monstrous Moonshine'. A few years ago, Eguchi, Ooguri and Tachikawa (EOT) made a similar observation regarding φ0,1: they noted that its Fourier coefficients can be interpreted in terms of representations of M24-representations, where M24 is the largest Mathieu group, another

simple sporadic finite group. Moreover, this observation has a nice formulation in terms of the mock modular form that is naturally associated to $\varphi_{0,1}$.

While the conjecture of EOT has by now been largely established, there are many intriguing open questions that remain. For example, while the Monster CFT provided a microscopic explanation of at least some aspects of Monstrous Moonshine, none of the superstring theories on K3 actually possess M24 as their automorphism group. Moreover, intriguing extensions of the EOT conjecture to higher index weak Jacobi forms (umbral moonshine) and generalisations to situations with less supersymmetry have recently been found, but the proper context in which all of these observations fit remains to be understood. In this program, we hope to bring together experts from different areas, including vertex operator algebra, string theory, algebraic geometry and number theory, in order to make progress with these very topical problems.