Notes on Bistafa navier stokes

On the development of the Navier-Stokes equation by Navier

despite its importance, only a few theoretical developments attempted to include the effects of viscosity in the equations of motion of fluids. One of these was proposed by Leonhard Euler (1707-1783) himself [4], in which he incorrectly assumed that, similar to the case of friction in solids, fluid friction was proportional to pressure [5]….

After Navier, the N-S equation was re-obtained by different arguments by a number of authors2. Nonetheless, Navier was never acknowledged for his contribution, as his approaches were not influential and were ignored by his successors and by specialized publications on the subject alike. At the time, though each new discoverer of the N-S equation seemed to have been aware of the derivation by Navier, they chose to ignore it….

The modern theory of elasticity may be considered to have its birth in 1821 when Navier first gave the equations for the equilibrium and motion of an (isotropic, one-constant) elastic solid. …

read at the Academy on 14 th May, 1821…

He soon perceived that these equations could be extended to other continuous media, and taking as a starting point the equations for elastic solids, he wrote in the 1 st memoir the equation for the motion of viscous fluids, substituting fluid particle velocities for elastic solid displacements, and the fluid viscosity constant (called ‘adherence constant’ by Navier) for the elastic solid constant. Other investigators such as Cauchy, Poisson, and Saint-Venant, presumably encouraged by Navier's publications, took the opportunity to offer the equation for viscous fluid motion from their equations of elasticity. From the case for a ‘non-elastic body’, and by assuming that the stress tensor is proportional to the rate of deformation tensor, Cauchy [7] obtained the equation of motion for viscous fluids given by Navier. By assuming that the stresses in a fluid are related to the fluid's rate of deformation, in the same manner that the stresses in a solid are related to strain, Poisson [8] obtained the N-S equation, with some additions to the pressure gradient term. Saint-Venant [9], in turn, thinking in terms of transverse pressure acting on the faces of the sliding fluid particles, obtained a stress tensor that yields the differential equations of Navier, Cauchy, and Poisson with one constant parameter. All of these 19 th century investigators tried to fill the gap between the rational fluid mechanics of the perfect non viscous fluid developed in the 18 th century by the Bernoullis (Daniel and Johann), d'Alembert, Euler, and Lagrange, and the actual behaviour of real viscous fluids in hydraulic systems….

decided to include internal fluid friction into the fundamental equations of hydrodynamics. By applying methods similar to Cauchy's and Poisson's, he arrived at the N-S equation by saying that this equation and the equation of continuity “[…] are applicable to the determination of the motion of water in pipes and canals, to the calculation of friction on the motions of tides and wave,…”

…The present work uncovers current concepts on the topic of viscous fluid flow that have Navier's imprint, but whose authorship has often been overlooked, taken for granted and other results that customarily have not been directly attributed to him. These new discoveries show that the impact of Navier's works on viscous flow was more profound than Navier scholars have so far proposed….

2. What is the Navier-Stokes equation?