

**Notes on:
Geometry and Quantum Mechanics**

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Abstract:

Attempts for a geometrical interpretation of Quantum Theory were made, notably the deBroglie-Bohm formulation. This was further refined by Santamato who invoked Weyl's geometry. However these attempts left a number of unanswered questions. In the present paper we return to these two formulations, in the context of recent studies invoking fuzzy spacetime and noncommutative geometry. We argue that it is now possible to explain the earlier shortcomings. At the same time we get an insight into the geometric origin of the deBroglie wavelength itself as also the Wilson-Sommerfeld Quantization rule.

The so called deBroglie-Bohm hydrodynamical formulation originated with Madelung and was developed by Bohm using deBroglie's pilot wave ideas. In this formulation, while the initial position coordinates in a Quantum Mechanical trajectory, are random, the trajectories themselves are determined by classical mechanics.

Bohm invoked a Quantum potential which is again related to the wave function. This mysterious quantum potential has been a stumbling block in the acceptance of the formulation.

By invoking Weyl's geometry, Santamato developed the deBroglie-Bohm formulation by relating the Quantum potential to the geometric structure of space; specifically to the curvature.

Unfortunately, Weyl's theory sought unsuccessfully to unify electromagnetism with gravitation and did not find favour.

Using noncommutative geometry, Sidharth attempts to show that the concept of fuzzy spacetime will resolve the issue.

The following is Sidharth's rendering of Bohm's version of quantum mechanics, from sidharth's paper:

Let us briefly review the above theory [7]. We start with the Schrodinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi \quad (1)$$

In (1), the substitution

$$\psi = R e^{iS/\hbar} \quad (2)$$

where R and S are real functions of \vec{r} and t , leads to,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (3)$$

$$\frac{1}{\hbar} \frac{\partial S}{\partial t} + \frac{1}{2m} (\vec{\nabla} S)^2 + \frac{V}{\hbar^2} - \frac{1}{2m} \frac{\nabla^2 R}{R} = 0 \quad (4)$$

where

$$\rho = R^2, \vec{v} = \frac{\hbar}{m} \vec{\nabla} S$$

and

$$Q \equiv -\frac{\hbar^2}{2m} (\nabla^2 R/R) \quad (5)$$

In Bohm's alternative formulation of Quantum Mechanics the definite value of position is a hidden variable while the quantum potential Q can be non local. These features are not popular with physicists.

Weyl postulated that in addition to the coordinate transformations of General Relativity, there were also gauge transformations which multiplied all components of the metric tensor $g_{\mu\nu}$, by an arbitrary function of the coordinates. So, the line elements would no longer be invariant. In its modern version, the metric tensor is normalized so that its determinant is given by [5],
[ie Q , the quantum potential ???]
 $|g_{\mu\nu}| = -1,$

In Weyl's geometry, even in a Euclidean space there is a non vanishing curvature R . Santamato exploits this fact, within the context of the deBroglie-Bohm theory, by postulating a Lagrangian minimum, which leads to the Hamilton-Jacobi and Schrodinger equations expressed in terms of the scalar curvature R .

According to Sidharth, the Quantum potential Q is now given in terms of the scalar curvature R .

It is the underlying fuzzyness of space time that brings out Weyl's geometry, not as an ad hoc feature, but as a truly geometrical consequence, and therefore legitimises Santamato's postulative approach of extending the deBroglie-Bohm formulation.

The deBroglie wavelength

$$\lambda = h/mV$$

follows from the noncommutative geometry of space time, rather than the physical Heisenberg Uncertainty Principle.

"This" is also the origin of the Wilson-Sommerfeld quantization rule, an otherwise mysterious Quantum Mechanical prescription.

Discussion

We have argued from the point of view of the noncommutative geometry, which, after many decades, is now coming to be recognized in the context of Quantum Superstring theory and Quantum Gravity.

It is well known that the so called Nelsonian stochastic process resembles the deBroglie-Bohm formulation, with very similar equations. However in the former case, both the position and velocity are not deterministic because of an underlying Brownian process.

It has been

shown by the author [12, 7], that both the similar approaches in fact can be unified for relativistic velocities, by considering quantized vortices originating from (11) of the order of the deBroglie, now the Compton scale λ . This immediately brings us back to the fuzzy noncommutative geometry (10). At the same time it must be pointed out that the supposedly unsatisfactory non local features of the Quantum potential Q become meaningful in the above context at the Compton scale, within which indeed we have exactly such non local effects [13].

It may be pointed out that more recently we have been led back to the background Quantum vacuum and the underlying Zero Point Field, now christened dark energy by the observation of the cosmological constant implied by the accelerated expansion of the universe [7] and it is this ZPF which provides the Brownian process of the stochastic theory. As pointed out by Nottale [14], such a Brownian process also eliminates the hidden variable feature of the deBroglie-Bohm formulation.

Interestingly it has been argued by Enz [15] that a particle extension, as is implied in the above considerations in the form of a quantized vortex or fuzzy space time, is the bridge between the particle and wave aspects. Infact at this scale, there is zitterbewegung, reminiscent of deBroglie's picture of a particle as an oscillator.

Originally Dirac had interpreted the zitterbewegung oscillations as unphysical, which are removed by the fact that due to the Uncertainty Principle we cannot go down to arbitrarily small space time intervals, so that our space time points are averages over Compton scale intervals (Cf.[7, 16] for a discussion).

3. It is also interesting to note that Santamato's tying up of the Quantum potential with the curvature R has been interpreted as being the result of cosmic fluctuations [17],