physics geometry Ali Nassar notes

There is more in the paper, but I don’t understand it well enough to dig it out.

Physics and Geometry
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original link:

http://users.aims.ac.za/~ali/Physics_Geometry.pdf this link is no longer working

This paper gives insight into B. G. Sidharth’s paper geometry and Quantum Mechanics

Abstract

Geometrical ideas have played a crucial role in the development of Physics. Einstein's General Theory of Relativity is a classic example of a theory which is fully geometric. It can be shown also that Gauge Field Theories have a deep geometric meaning, where the potentials of the gauge field play the role of a *connection* over some fibre bundles, and the gauge field associated with this potential is simply the curvature of the connection.

Gravitation is described as a purely geometric effect in Einstein’s theory of Relativity through his equation

$G_{\mu\nu}$ = 8 pi G $T_{\mu\nu}$

where $G_{\mu\nu}$ is a geometric object describing the geometry of space-time and $T_{\mu\nu}$ is the energy-momentum tensor.

One of the consequences of the space-time being curved is the introduction of another derivative, the covariant derivative, which reduces to the ordinary derivative in the limit of a flat space-time.

The idea of gauging was first proposed by Weyl while he was trying to incorporate electromagnetism into geometry using space-time dependent (local) scale transformations.

The idea of gauge transformations emerged again after the advent of modern quantum mechanics. The fundamental equations of quantum mechanics are invariant under the transformation of the wave function by $e^{ia:}$:

Psi -> e^{ia} Psi where a is an arbitrary constant phase

If the phase transformation is space-time dependent; ie, local, the form will be:

Psi(x) -> $^{eia(x)}$ Psi(x)

This (one-parameter) Abelian group of unitary transformations is called U(1). The transformations represent rotations in the internal space of the fields. These rotations are space-time dependent, i.e., the rotation angle will change from one point to another in space-time. These rotations have nothing to do with ordinary rotations in 3-D space.

Our discussion so far considered only the simplest case of a one-parameter group U(1) which is the gauge group of electromagnetism. The extension to include other gauge fields is done via Yang-Mills theory.

Gauge groups in the Yang-Mills theory are no longer one-parameter (hence, non-Abelian) and the fibre bundle structure will be more rich.

GTR is another example of a gauge theory where the gauge group is the group of all diffeomorphisms on the space-time manifold. The study of the connections between space-time symmetries and gauge symmetries may in the end lead to a unified theory for all the fundamental interactions of nature.