Linear Systems Theory

- Introduction – Receptive fields and mechanisms
- Fourier Analysis – Signals as sums of sine waves
- Linear, shift-invariant systems
  - Definition
  - Applied to impulses, sums of impulses
  - Applied to sine waves, sums of sine waves
- Applications

Fourier Analysis

Signals as sums of sine waves

- 1-d: time series
  - fMRI signal from a voxel or ROI
  - mean firing rate of a neuron over time
  - auditory stimuli
- 2-d: static visual image, neural image
- 3-d: visual motion analysis
- 4-d: raw fMRI data

Linear Systems Analysis

Systems with signals as input and output

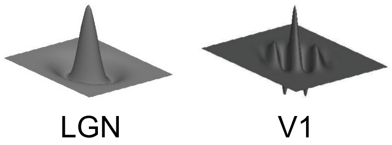
- 1-d: low- and high-pass filters in electronic equipment, fMRI data analysis, or in sound production (articulators) or audition (the ear as a filter)
- 2-d: optical blur, spatial receptive field
- 3-d: spatio-temporal receptive field

Spatial Vision

- Image representation or coding
  - At each stage, what information is kept and what is lost?
- Image analysis
- Nonlinear: pattern recognition

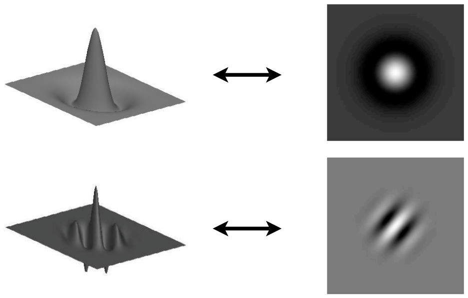
Receptive Field

- In any modality: that region of the sensory apparatus that, when stimulated, can directly affect the firing rate of a given neuron
- Spatial vision: spatial receptive field can be mapped in visual space or on the retina
- Examples:



Receptive Field

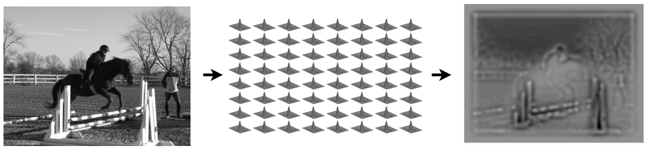
A spatial receptive field is an image



with its own Fourier transform.

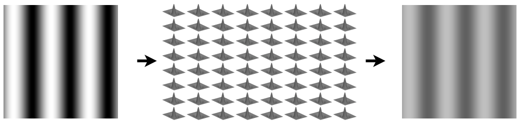
Neural Image

A spatial receptive field may also be treated as a linear system, by assuming a dense collection of neurons with the same receptive field translated to different locations in the visual field:



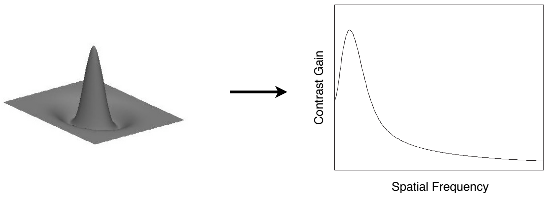
Neural Image of a Sine Wave

For a linear, shift-invariant system such as a linear model of a receptive field, an input sine wave results in an identical output sine wave, except for a possible lateral shift and scaling of contrast.



Frequency Response

This scaling of contrast by a linear receptive field in the neural image is a function of spatial frequency determined by the shape of the receptive field.



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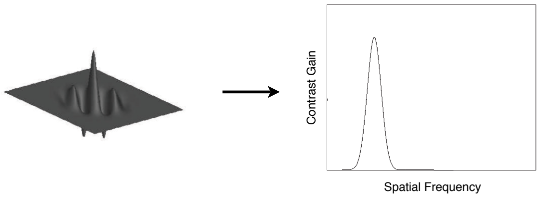
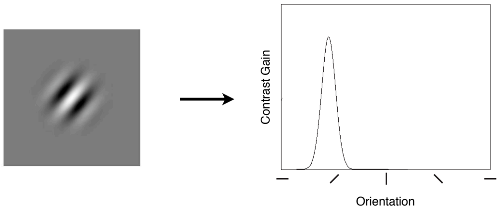
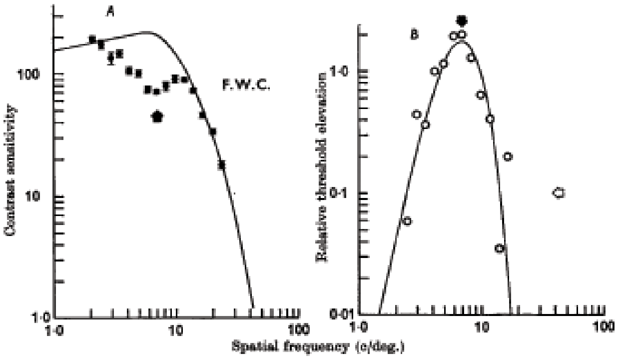


Orientation Tuning

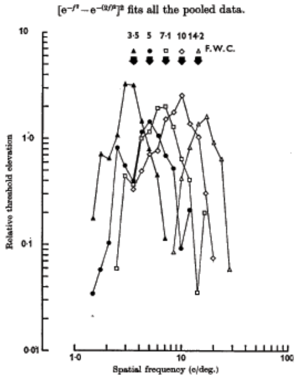
If a receptive field is not circularly symmetric, the scaling of contrast is also a function of orientation (for a given spatial frequency) determined by the shape of the receptive field.



Application Preview: SF Adaptation (Blakemore & Campbell, 1969)



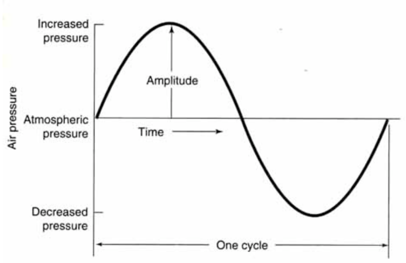
Application Preview: SF Adaptation (Blakemore & Campbell, 1969)



Summary: Linear Systems Theory

- Signals can be represented as sums of sine waves
- Linear, shift-invariant systems operate “independently” on each sine wave, and merely scale and shift them.
- A simplified model of neurons in the visual system, the linear receptive field, results in a neural image that is linear and shift-invariant.
- Psychophysical models of the visual system might be built of such mechanisms.
- It is therefore important to understand visual stimuli in terms of their spatial frequency content.
- The same tools can be applied to other modalities (e.g., audition) and other signals (EEG, MRI, MEG, etc.).

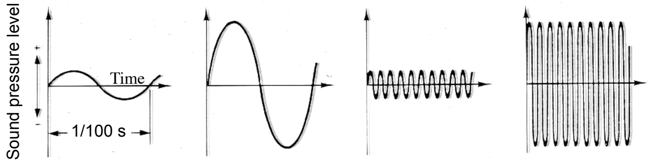
Auditory example: Pure tones



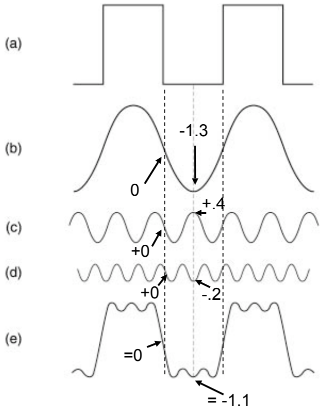
Pure tones can be described by 3 numbers:
Frequency = rate of air pressure modulation (related to pitch)
Amplitude = sound pressure level (related to loudness)
Phase = sin vs. cosine vs. another horizontal shift

Frequency and amplitude

weak 100 Hz
strong 100 Hz
weak 1000 Hz
strong 1000 Hz

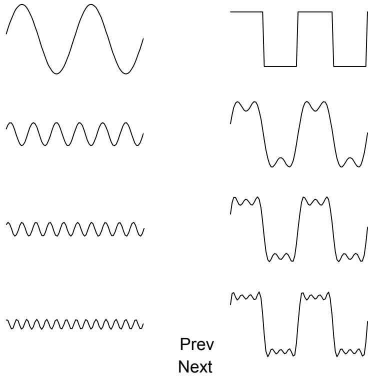


Fourier components of a square wave



Fourier components of a square wave

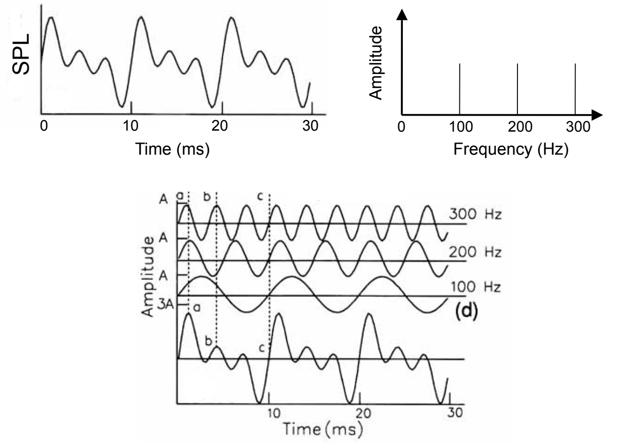
Ampl.	Freq.
1	100
1/3	300
1/5	500
1/7	700



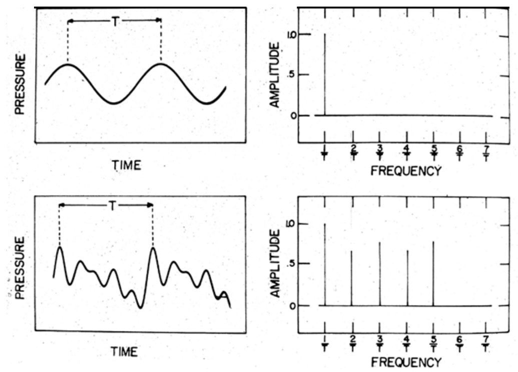
Square wave
‘f’ + ‘3f’/3
‘f’ + ‘3f’/3+’5f’/5
‘f’ + ‘3f’/3+’5f’/5+’7f’/7

Prev
Next

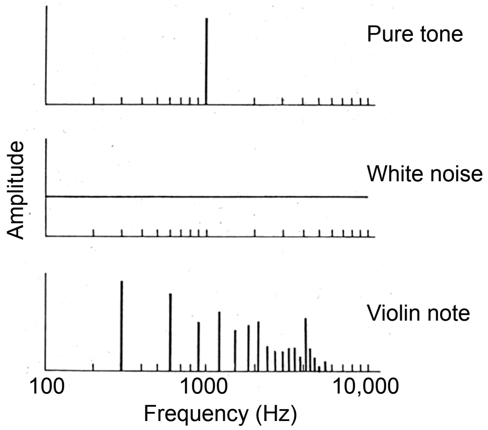
Fourier Synthesis – Building Stimuli from Sine Waves



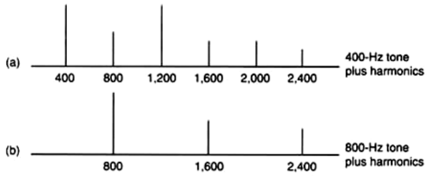
Fourier spectrum representation of sound



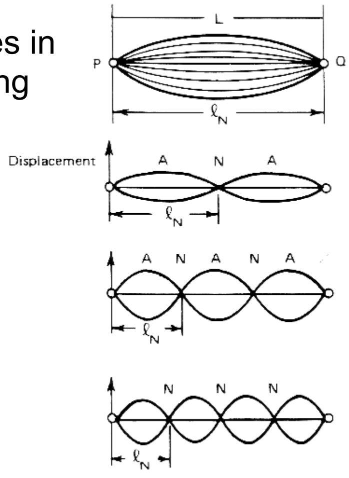
Fourier spectra of some sounds



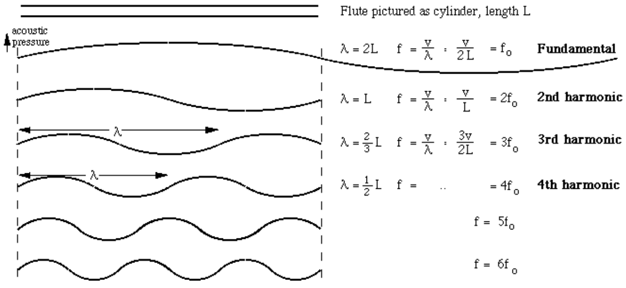
Fundamental frequency and harmonics



Standing waves in a vibrating string



Flute (open pipe) harmonics



Flute pictured as cylinder, length L

$\lambda = 2L \quad f = \frac{v}{\lambda} : \frac{v}{2L} = f_o$ **Fundamental**

$\lambda = L \quad f = \frac{v}{\lambda} : \frac{v}{L} = 2f_o$ **2nd harmonic**

$\lambda = \frac{2}{3}L \quad f = \frac{v}{\lambda} : \frac{3v}{2L} = 3f_o$ **3rd harmonic**

$\lambda = \frac{1}{2}L \quad f = \quad .. \quad = 4f_o$ **4th harmonic**

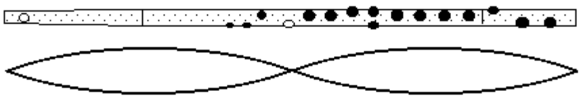
$f = 5f_o$

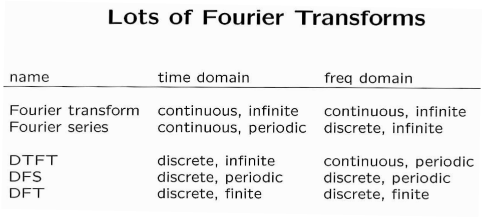
$f = 6f_o$

Flute (open pipe) harmonics

Other notes (shorten the pipe)

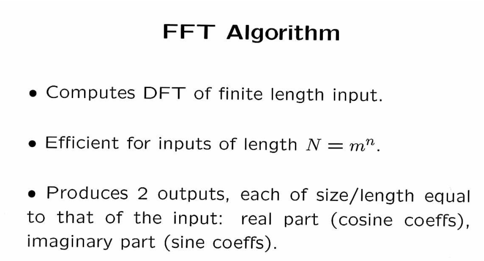
Reinforcing a harmonic (forcing a “node”)





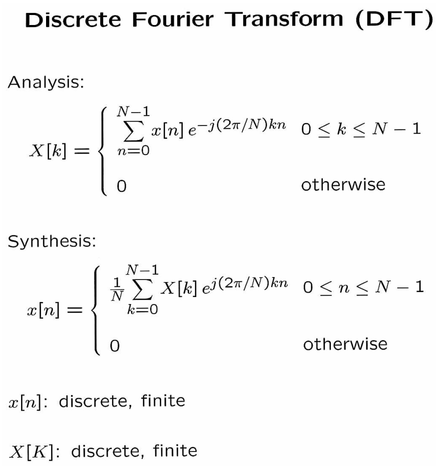
Lots of Fourier Transforms

name	time domain	freq domain
Fourier transform	continuous, infinite	continuous, infinite
Fourier series	continuous, periodic	discrete, infinite
DTFT	discrete, infinite	continuous, periodic
DFS	discrete, periodic	discrete, periodic
DFT	discrete, finite	discrete, finite



FFT Algorithm

- Computes DFT of finite length input.
- Efficient for inputs of length $N = m^n$.
- Produces 2 outputs, each of size/length equal to that of the input: real part (cosine coeffs), imaginary part (sine coeffs).



Discrete Fourier Transform (DFT)

Analysis:

$$X[k] = \begin{cases} \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn} & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

Synthesis:

$$x[n] = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{j(2\pi/N)kn} & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

$x[n]$: discrete, finite

$X[K]$: discrete, finite

Complex numbers and complex exponentials

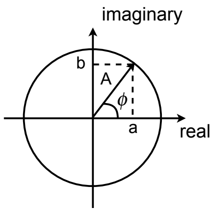
$$z = a + bi = Ae^{i\phi} = A\left[\cos\phi + i\sin\phi\right]$$ where i [or j] $= \sqrt{-1}$

real part imaginary part amplitude phase

where:

$$a = A\cos\phi \qquad b = A\sin\phi$$

$$A = \sqrt{a^2 + b^2} \qquad \phi = \tan^{-1}(b/a)$$



Why bother?

$$\left(A_1 e^{i\phi_1}\right)\left(A_2 e^{i\phi_2}\right) = A_1 A_2 e^{i(\phi_1 + \phi_2)}$$

amplitudes multiply phases add

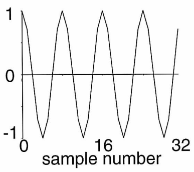
Discrete Fourier Transform

$$X\left[k\right] = \sum_{n=0}^{N-1} x\left[n\right] e^{-i(2\pi/N)kn} \quad 0 \le k \le N-1$$

$$= \sum_{n=0}^{N-1} x\left[n\right]\left(\cos((2\pi/N)kn) + i\sin((2\pi/N)kn)\right)$$

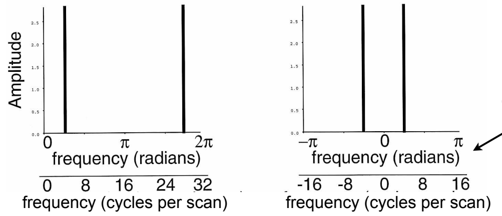
k is frequency in cycles/image (or cycles/signal) and is computed effectively only for frequencies zero (DC), 1, 2, ..., N/2. The vector you get back from MATLAB (fft or fft2, inverses are ifft and ifft2), however, continues redundantly (for real signals, that is): 0, 1, ..., N/2-1, N/2=0, N/2-1, ..., 1.

DFT of a Cosinusoid



$\cos(\frac{2\pi kx}{N})$

k = 4 cycles/scan
N = 32 time points



$2\pi k / N$
(π / 4 in this case)

DFT of a Sine Wave

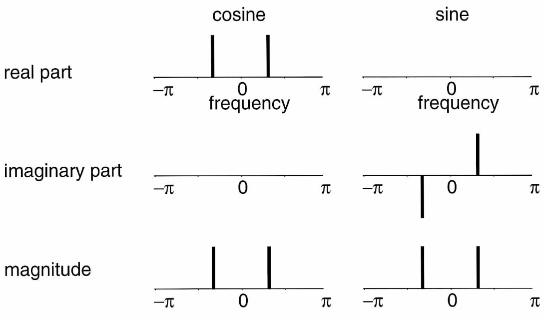
$$A\cos\left(\frac{2\pi kn}{N}-\phi\right)=A\cos\phi\cos\frac{2\pi kn}{N}+A\sin\phi\sin\frac{2\pi kn}{N}$$

The Fourier coefficient for this frequency of *k* cycles/signal is:

$$a+bi=(A\cos\phi)+(A\sin\phi)i$$

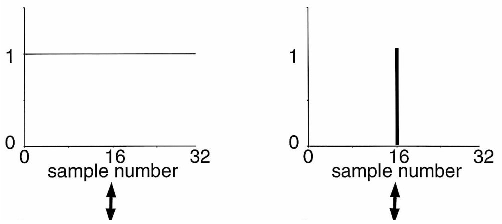
In other words, the amplitude is *A* and the phase is ϕ.

Real and Imaginary Parts



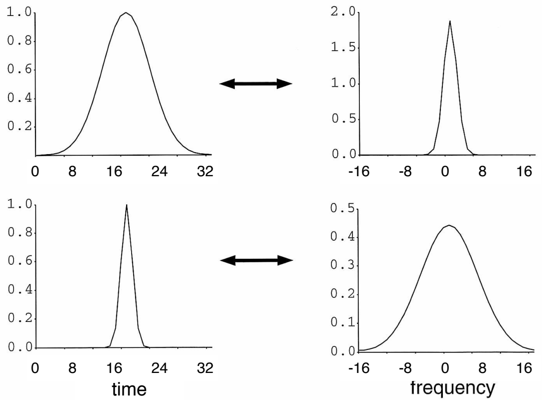
phase 0 π/2

DFT of Impulse Signals and Constant Signals



?

Uncertainty Principle

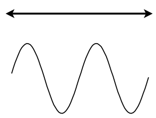


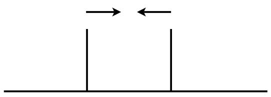
Similarity Theorem

If $f(x)\leftrightarrow F(\omega)$

then $f(ax)\leftrightarrow\frac{1}{|a|}F\left(\frac{\omega}{a}\right)$

If you stretch the *x*-axis, you shrink the frequency axis



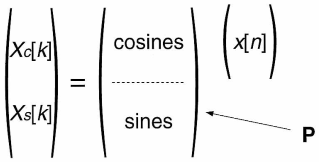


Discrete Fourier Transform Matrix

Analysis: $X[k] = \sum_n x[n] \exp(-j2\pi kn/N)$

For real valued inputs:

$X_c[k] = \sum_n x[n] \cos(...)$ $X_s[k] = \sum_n -x[n] \sin(...)$

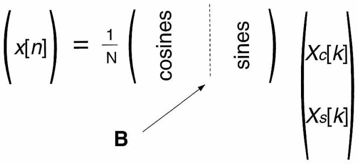


$$\begin{pmatrix} X_c[k] \\ \\ X_s[k] \end{pmatrix} = \begin{pmatrix} \text{cosines} \\ \hdashline \text{sines} \end{pmatrix} \begin{pmatrix} x[n] \end{pmatrix}$$

Rows of **P** called projection functions: $\frac{1}{N}\mathbf{P}^T\mathbf{P} = \mathbf{I}$

Discrete Fourier Transform Matrix

Synthesis: $x[n] = \sum_k X[k] \exp(j2\pi kn/N)$

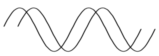


$$\begin{pmatrix} x[n] \end{pmatrix} = \frac{1}{N} \begin{pmatrix} \text{cosines} & \vdots & \text{sines} \end{pmatrix} \begin{pmatrix} X_c[k] \\ \\ X_s[k] \end{pmatrix}$$

Cols of **B** called basis functions. $\mathbf{B} = \mathbf{P}^T$, $\frac{1}{N}\mathbf{B}\,\mathbf{B}^T = \mathbf{I}$

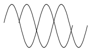
Properties of the DFT

Circular shift



f, 90 deg shift

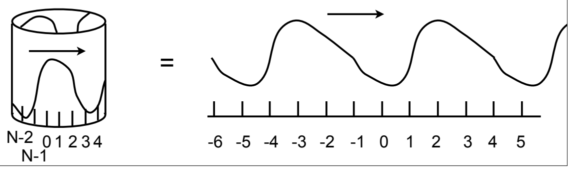
If x[n] ⟷ X[k], then:



2f, 180 deg shift

$x[(n-m)_N] \longleftrightarrow \exp(-j2\pi km/N)\, X[k]$

i.e., only phase (not magnitude) is affected



Time/Space Reversal

If $x[n] \Leftrightarrow X[k]$,
then $x[-n] = x[N-n] \Leftrightarrow X^*[k]$
where * means complex conjugate.

Analysis eqn:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn}$$

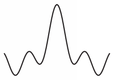
Change of variables ($m = N-n$, $n = N-m$):

$$\begin{aligned} \sum_{n=0}^{N-1} x[N-n]\, e^{-j(2\pi/N)kn} &= \sum_{m=N}^{m=1} x[m]\, e^{j(2\pi/N)km}\, e^{-j2\pi k} \\ &= \sum_{m=0}^{N-1} x[m]\, e^{j(2\pi/N)km} \end{aligned}$$

For real $x[m]$,

$$\sum_{m=0}^{N-1} x[m]\, e^{j(2\pi/N)km} = X^*[k]$$

Even/Odd-Symmetric Signals



Even signal: $x[-n] = x[n]$

Odd signal: $x[-n] = -x[n]$

Any signal can be decomposed:

$$x[n] = x_e[n] + x_o[n]$$

where x_e is even,

$$x_e[n] = \tfrac{1}{2}(x[n] + x[-n])$$

and x_o is odd,

$$x_o[n] = \tfrac{1}{2}(x[n] - x[-n])$$

DFT of Even/Odd Signals

DFT of even signal is real:

$$\begin{aligned} x_e[n] &= \tfrac{1}{2}(x_e[n] + x_e[-n]) \\ X_e[k] &= \tfrac{1}{2}(X_e[k] + X_e^*[k]) \\ &= \text{Re}\{X_e[k]\} \end{aligned}$$

i.e., all cosine terms

DFT of odd signal is imaginary:

$$\begin{aligned} x_o[n] &= \tfrac{1}{2}(x_o[n] - x_o[-n]) \\ X_o[k] &= \tfrac{1}{2}(X_o[k] - X_o^*[k]) \\ &= j\text{Im}\{X_o[k]\} \end{aligned}$$

i.e., all sine terms

Symmetry Properties

For real $x[n]$:

1. Re$\{X[k]\}$ is even.
2. Im$\{X[k]\}$ is odd.
3. $||X[k]||$ is even.
4. Phase of $X[k]$ is odd.

To generate a signal or image from a transform: Adopt the symmetry constraints and inverse transform $\Rightarrow$ real signal.

Parseval's Theorem

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$

Derivation for real $x[n]$:

$$X[k] = \mathbf{P}x[n] \quad \frac{1}{N}\mathbf{P}^T\mathbf{P} = \mathbf{I}$$

$$\begin{aligned} \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2 &= \frac{1}{N}X^T[k]X[k] \\ &= \frac{1}{N}(\mathbf{P}x[n])^T(\mathbf{P}x[n]) \\ &= \frac{1}{N}(x^T[n]\mathbf{P}^T\mathbf{P}x[n]) \\ &= x^T[n]\left(\tfrac{1}{N}\mathbf{P}^T\mathbf{P}\right)x[n] \\ &= x^T[n]x[n] \end{aligned}$$

Intuition: $\mathbf{P}$ is orthogonal (orthonormal when multiply by $1/N$) like a rotation matrix. Length is preserved under rotation.

"Orthonormal" DFT

Analysis:

$$X[k] = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn}$$

Synthesis:

$$x[n] = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X[k]\, e^{j(2\pi/N)kn}$$

In matrix notation:

$$\begin{aligned} X[k] &= \mathbf{P}x[n] \\ x[n] &= \mathbf{B}X[k] \end{aligned}$$

where

$$\begin{aligned} \mathbf{P}^T\mathbf{P} &= \mathbf{I} \\ \mathbf{B} &= \mathbf{P}^T \end{aligned}$$

Other Properties of the Fourier Transform

Linearity $\quad f + g \leftrightarrow F + G$

Derivative $\quad \dfrac{d\cos 2\pi\omega x}{x} = -2\pi\omega\sin 2\pi\omega x, \quad \dfrac{d\sin 2\pi\omega x}{x} = 2\pi\omega\cos 2\pi\omega x$

Hence: $\dfrac{\text{df}}{\text{dx}} \leftrightarrow -i2\pi\omega F$

Integral $\quad \int f\,dx \leftrightarrow \dfrac{iF}{2\pi\omega}$

DTFT and DFS

Discrete-Time Fourier Transform (DTFT)

Analysis:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

Synthesis:

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$x[n]$: discrete, infinite, not necessarily periodic

$X(\omega)$: continuous, periodic (with period 2π)

$X(\omega)$ is Periodic

$$\begin{aligned} X(\omega + 2\pi) &= \sum x[n]\, e^{-j(\omega+2\pi)n} \\ &= \sum x[n]\, e^{-j\omega n} e^{-j2\pi n} \\ &= \sum x[n]\, e^{-j\omega n} \\ &= X(\omega) \end{aligned}$$

where:

$$\begin{aligned} e^{-j2\pi n} &= \cos(2\pi n) + j\sin(2\pi n) \\ &= 1 + 0 \end{aligned}$$

DFT versus DTFT

DFT is equally spaced samples of DTFT.

DTFT for $x[n]$ of finite length, i.e., zero outside $[0, N-1]$:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{N-1} x[n]\, e^{-j\omega n}$$

DFT:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j(2\pi/N)kn}$$

Same for $\omega = (2\pi/N)k$, that is, for N equally spaced freqs between $\omega = 0$ and $\omega = 2\pi$.

Discrete Fourier Series (DFS)

Analysis:

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n]\, e^{-j(2\pi/N)kn}$$

Synthesis:

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k]\, e^{j(2\pi/N)kn}$$

$\tilde{x}[n]$: discrete, periodic (period N)

$\tilde{X}[k]$: discrete, periodic (period N)

DFS versus DFT

$\tilde{X}[k]$ & $\tilde{x}[n]$ are periodic extensions of $X[k]$ & $x[n]$, e.g.,

$$\tilde{x}[n] = x[(n \bmod N)]$$

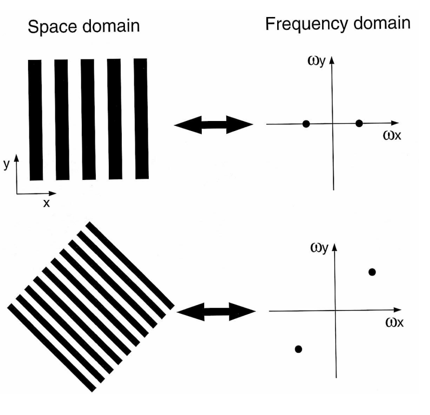
$$x[n] = \begin{cases} \tilde{x}[n] & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

The fact that $x[n]$ and $X[k]$ are zero outside $[0, N-1]$ is implied but not always stated.

Evaluating the analysis/synthesis equation outside $[0, N-1]$ does not give 0, but rather gives the periodic extension.

Edge effects:  vs. 

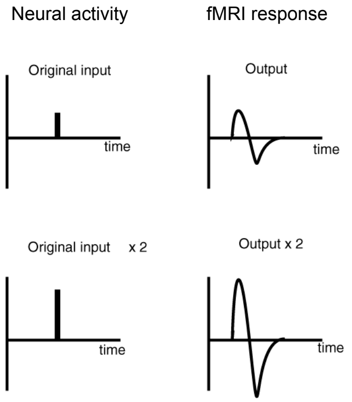
Two-dimensional Fourier transform



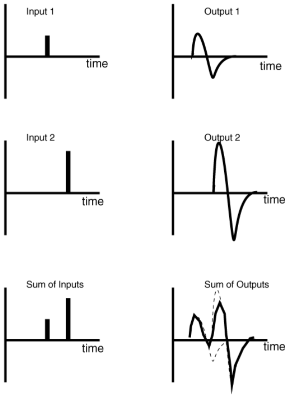
Linear, Shift-Invariant Systems

- Linearity: scalar rule and additivity
- Applied to impulse, sums of impulses
- Applied to sine waves, sums of sine waves

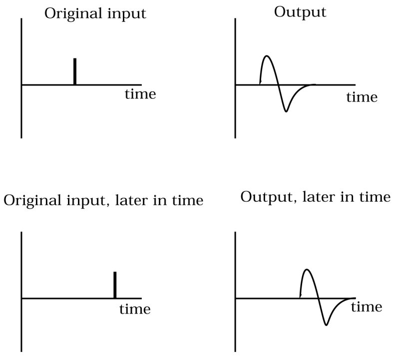
Homogeneity (scalar rule)



Additivity



Shift invariance



Linear systems

A system (or transform) converts (or maps) an input signal into an output signal:

$y(t) = T[x(t)]$

A linear system satisfies the following properties:

1) Homogeneity (scalar rule):
$T(a\, x(t)] = a\, y(t)$
2) Additivity:
$T(x_1(t) + x_2(t)] = y_1(t) + y_2(t)$

Often, these two properties are written together and called superposition:

$T(a\, x_1(t) + b\, x_2(t)] = a\, y_1(t) + b\, y_2(t)$

Shift invariance

For a system to be shift-invariant (or time-invariant) means that a time-shifted version of the input yields a time-shifted version of the output:

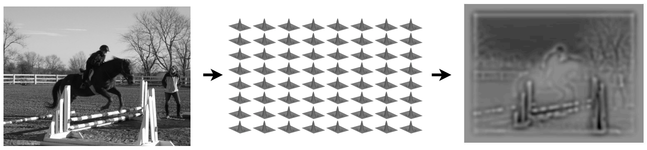
$y(t) = T[x(t)]$

$y(t - s) = T[x(t - s)]$

The response $y(t - s)$ is identical to the response $y(t)$, except that it is shifted in time.

Neural Image - Reprise

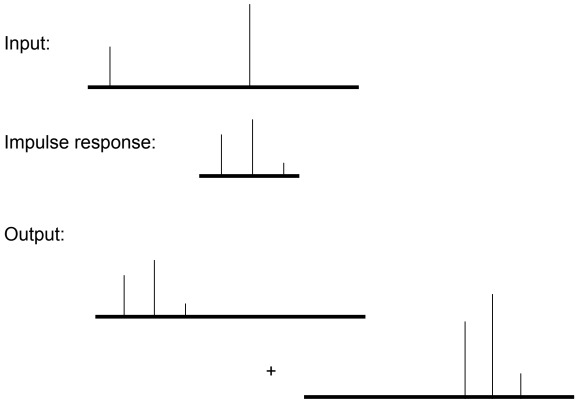
A spatial receptive field may also be treated as a linear system, by assuming a dense collection of neurons with the same receptive field translated to different locations in the visual field. In this view, it is a linear, shift-invariant system.



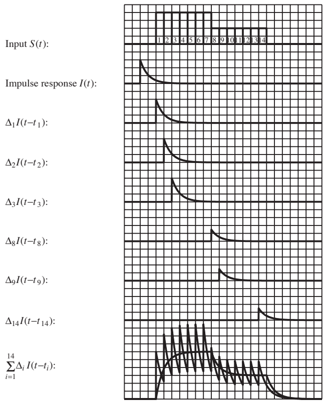
Linear, Shift-Invariant Systems

- Linearity: Scalar rule and additivity
- Applied to impulse, sums of impulses
- Applied to sine waves, sums of sine waves

Convolution as sum of impulse responses



Convolution as sum of impulse responses



Convolution

Discrete-time signal: $x[n] = [x_1, x_2, x_3, \ldots]$

A system or transform maps an input signal into an output signal:

$$y[n] = T\{x[n]\}$$

A shift-invariant, linear system can always be expressed as a convolution:

$$y[n] = \sum x[m]\, h[n-m]$$

where $h[n]$ is the impulse response.

Convolution derivation

Homogeneity:

$$T\{a\, x[n]\} = a\, T\{x[n]\}$$

Additivity:

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\}$$

Superposition:

$$T\{a\, x_1[n] + b\, x_2[n]\} = a\, T\{x_1[n]\} + b\, T\{x_2[n]\}$$

Shift-invariance:

$$y[n] = T\{x[n]\} \Rightarrow y[n-m] = T\{x[n-m]\}$$

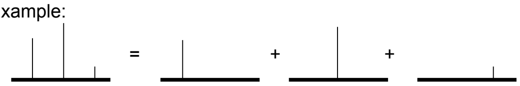
Convolution derivation (contd.)

Impulse sequence:
d[*n*] = 1 for *n* = 0, *d*[*n*] = 0 otherwise

Any sequence can be expressed as a sum of impulses:
$x[n] = \sum x[m]\, d[n-m]$

where
d[*n-m*] is impulse shifted to sample *m*
x[*m*] is the height of that impulse

Example:



Convolution derivation (cont)

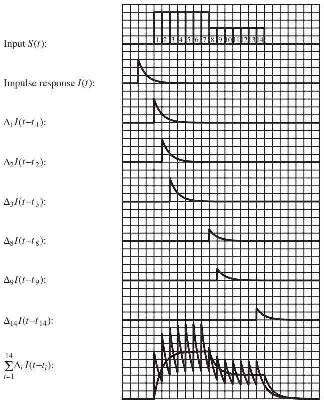
x[n]: input
y[n] = T{x[n]}: output
h[n] = T{d[n]}: impulse response

1) Represent input as sum of impulses:
$y[n] = T\{x[n]\}$
$y[n] = T\left\{\sum x[m]\, d[n-m]\right\}$

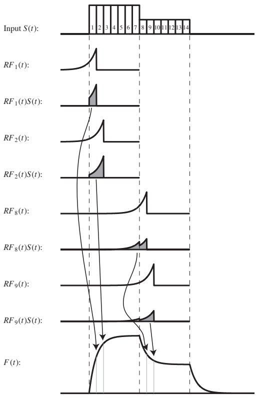
2) Use superposition:
$y[n] = \sum x[m]\, T\{d[n-m]\}$

3) Use shift-invariance:
$y[n] = \sum x[m]\, h[n-m]$

Convolution as sum of impulse responses



Convolution as correlation with the "receptive field" (time-reversed impulse response):



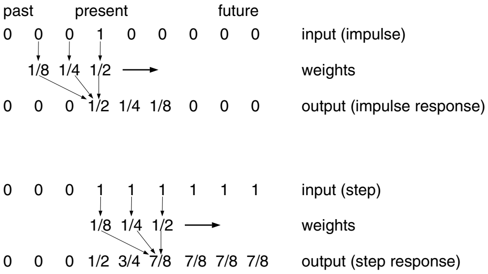
Convolution as matrix multiplication

Linear system <=> matrix multiplication
Shift-invariant linear system <=> Toeplitz matrix

$$
\begin{bmatrix} \vdots \\ 5 \\ 2 \\ -3 \\ 4 \\ \vdots \end{bmatrix}
=
\begin{bmatrix}
 & & & \vdots & & & & \\
\cdots & 1 & 2 & 3 & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 1 & 2 & 3 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 1 & 2 & 3 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 1 & 2 & 3 & \cdots \\
 & & & \vdots & & & &
\end{bmatrix}
\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ -1 \\ 2 \\ \vdots \end{bmatrix}
$$

Columns contain shifted copies of the impulse response.
Rows contain time-reversed copies of impulse response.

Convolution as sequence of weighted sums



past		present					future		
0	0	0	1	0	0	0	0	0	input (impulse)
	1/8	1/4	1/2	⟶					weights
0	0	0	1/2	1/4	1/8	0	0	0	output (impulse response)
0	0	0	1	1	1	1	1	1	input (step)
			1/8	1/4	1/2	⟶			weights
0	0	0	1/2	3/4	7/8	7/8	7/8	7/8	output (step response)

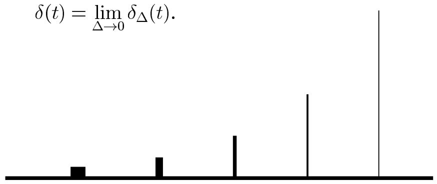
Continuous-time derivation of convolution

Pulses and impulses

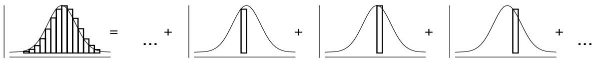
$$\delta(t) = \begin{cases} \infty & \text{if} \quad t = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & \text{if} \quad 0 < t < \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t).$$



Staircase approximation to continuous-time signal



$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t - k\Delta)\,\Delta.$$

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t - k\Delta)\,\Delta.$$

$$x(t) = \int_{-\infty}^{\infty} x(s)\,\delta(t - s)\,ds.$$

Convolution

Representing the input signal as a sum of pulses:

$$\begin{aligned} y(t) = T[x(t)] &= T\left[\int_{-\infty}^{\infty} x(s)\,\delta(t-s)\,ds\right] \\ &= T\left[\lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\,\delta_\Delta(t - k\Delta)\,\Delta\right]. \end{aligned}$$

Using additivity,

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} T[x(k\Delta)\,\delta_\Delta(t - k\Delta)\,\Delta].$$

Taking the limit,

$$y(t) = \int_{-\infty}^{\infty} T[x(s)\,\delta(t - s)\,ds].$$

Using homogeneity (scalar rule),

$$y(t) = \int_{-\infty}^{\infty} T[x(s)\,\delta(t - s)\,ds].$$

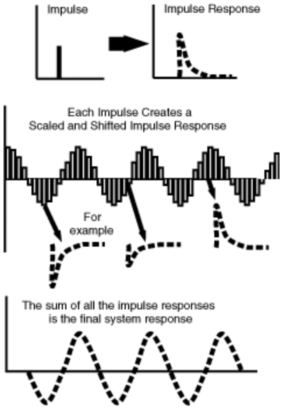
Defining *h*(*t*) as the impulse response,

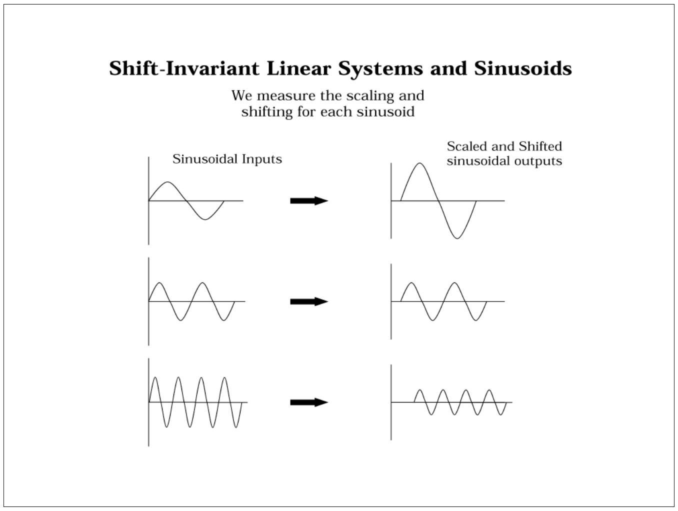
$$y(t) = \int_{-\infty}^{\infty} x(s)\,h(t - s)\,ds.$$

Linear, Shift-Invariant Systems

- Linearity: Scalar rule and additivity
- Applied to impulse, sums of impulses
- Applied to sine waves, sums of sine waves

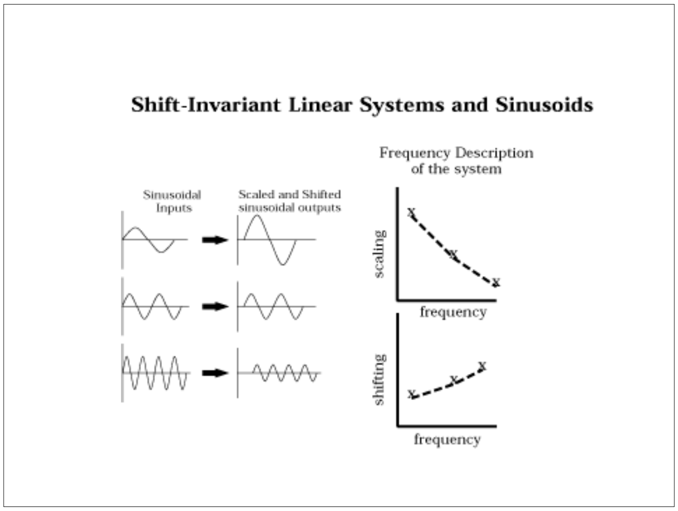
Shift-invariant linear systems and impulses



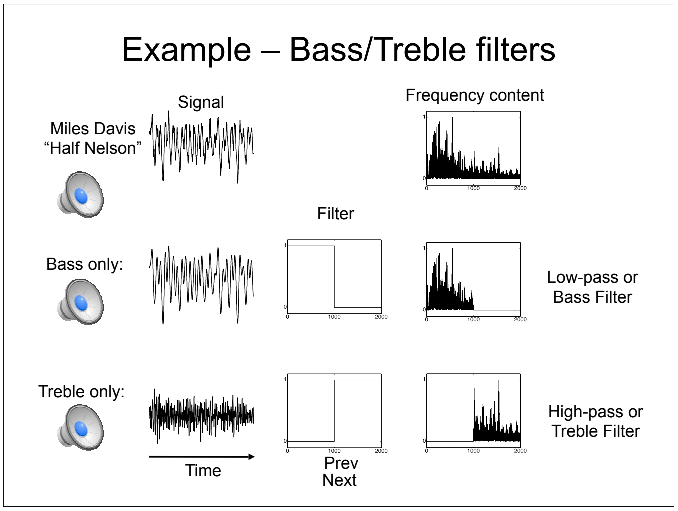


Shift-Invariant Linear Systems and Sinusoids

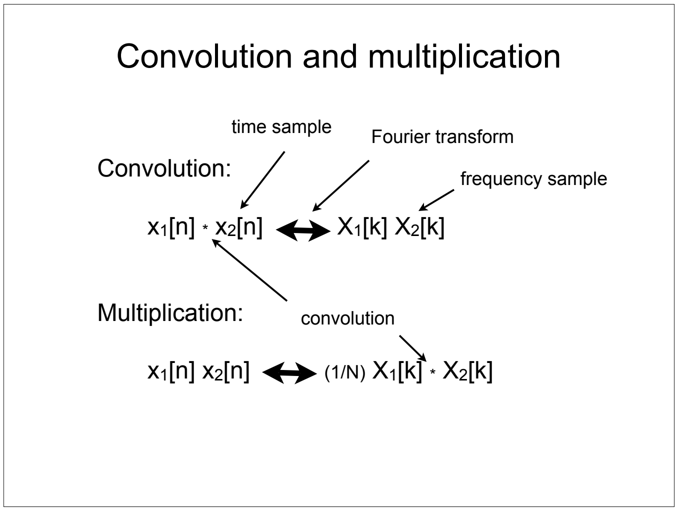
We measure the scaling and
shifting for each sinusoid



Shift-Invariant Linear Systems and Sinusoids



Example – Bass/Treble filters



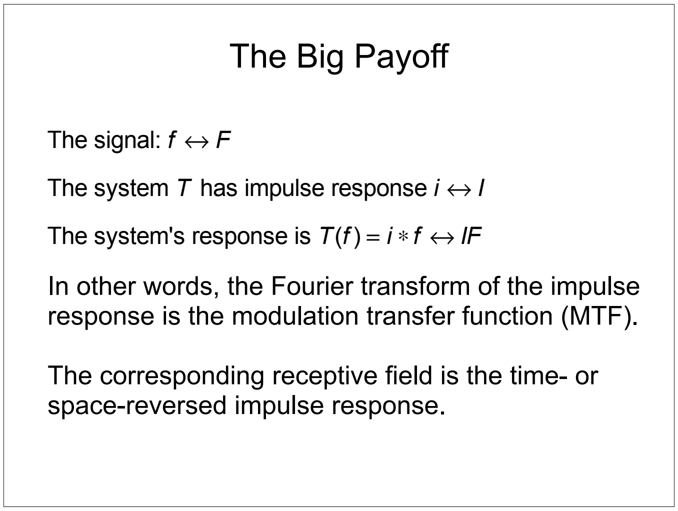
Convolution and multiplication

Convolution:

$x_1[n] * x_2[n] \longleftrightarrow X_1[k]\, X_2[k]$

Multiplication:

$x_1[n]\, x_2[n] \longleftrightarrow (1/N)\ X_1[k] * X_2[k]$



The Big Payoff

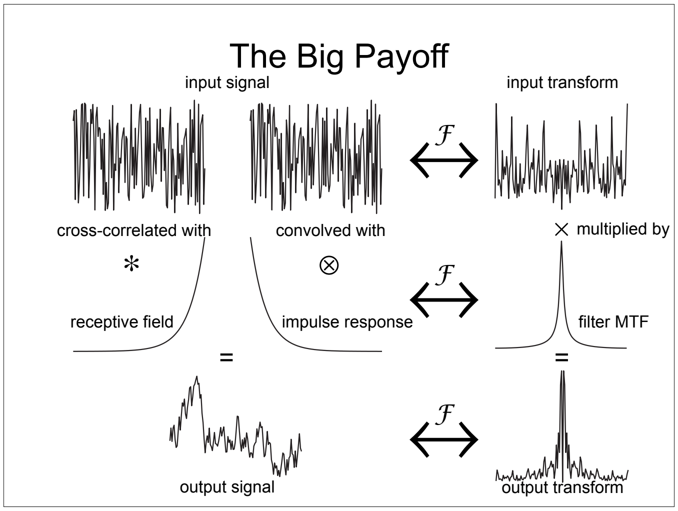
The signal: $f \leftrightarrow F$

The system T has impulse response $i \leftrightarrow I$

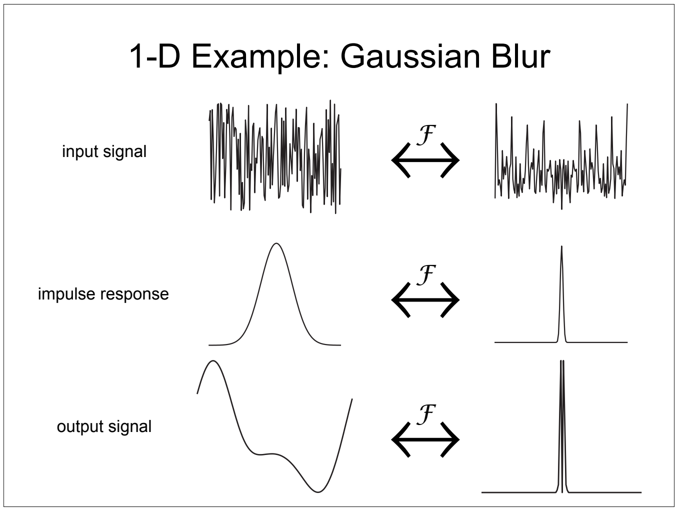
The system's response is $T(f) = i * f \leftrightarrow IF$

In other words, the Fourier transform of the impulse response is the modulation transfer function (MTF).

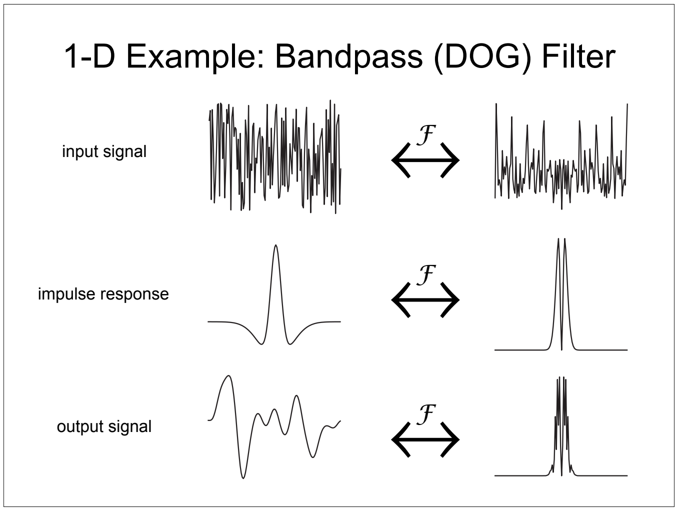
The corresponding receptive field is the time- or space-reversed impulse response.



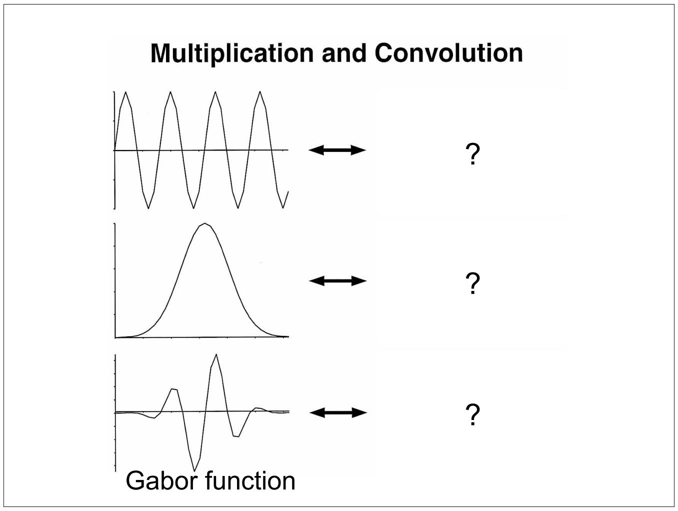
The Big Payoff



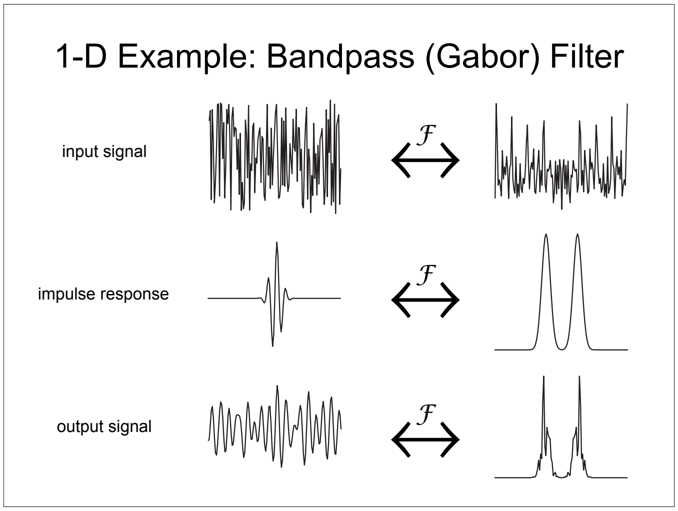
1-D Example: Gaussian Blur



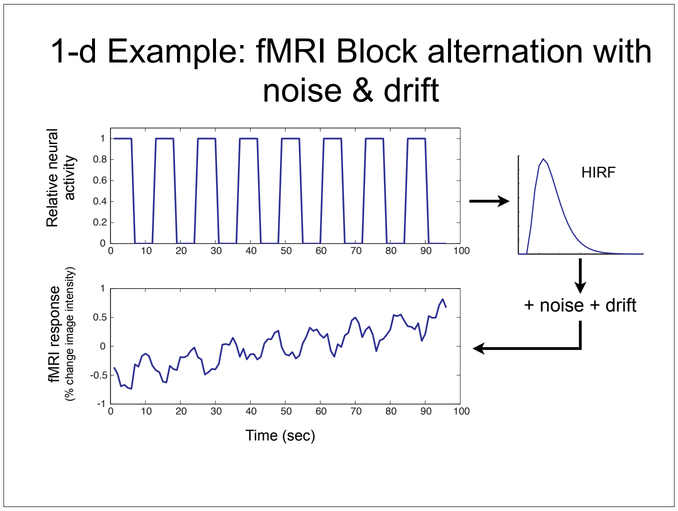
1-D Example: Bandpass (DOG) Filter



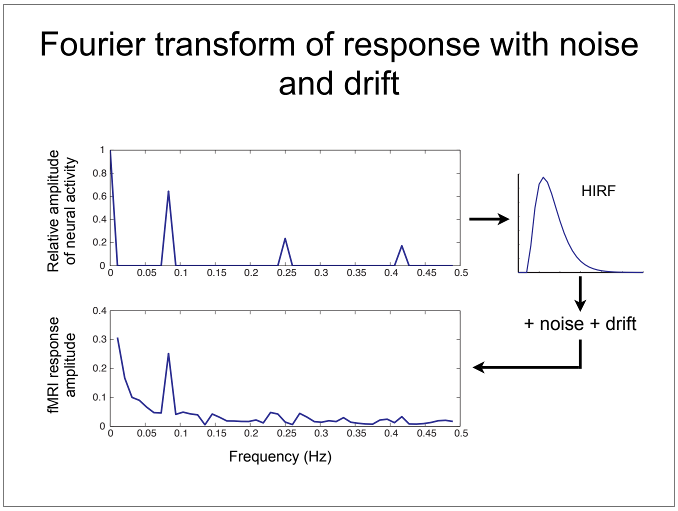
Multiplication and Convolution



1-D Example: Bandpass (Gabor) Filter



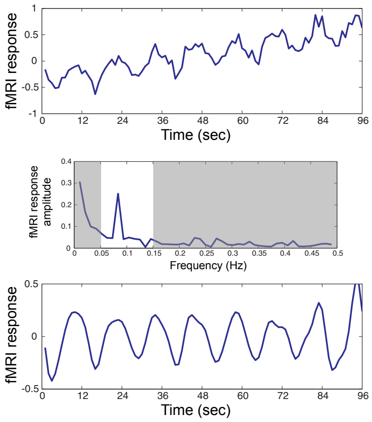
1-d Example: fMRI Block alternation with noise & drift



Fourier transform of response with noise and drift

Bandpass filtering to remove noise and drift

- How do you make a low-pass filter?
- How do you make a high-pass filter?

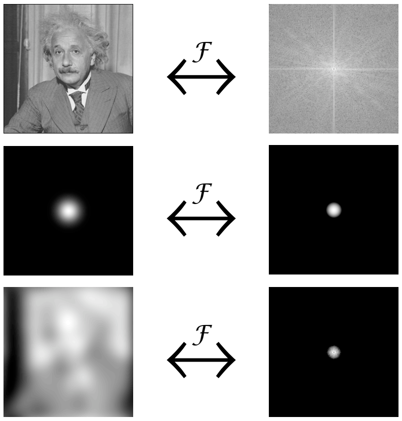


2-D Example: Gaussian Blur

input signal

impulse response

output signal

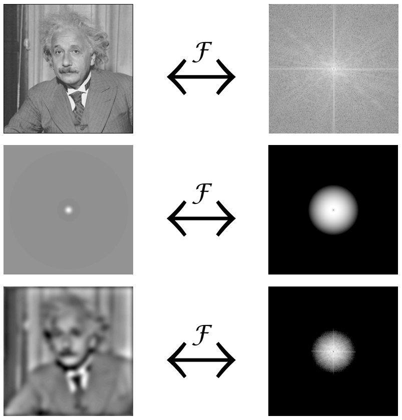


2-D Example: Bandpass (DOG) Filter

input signal

impulse response

output signal

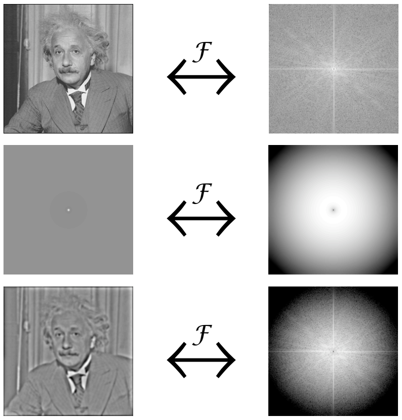


2-D Example: Bandpass (DOG) Filter

input signal

impulse response

output signal

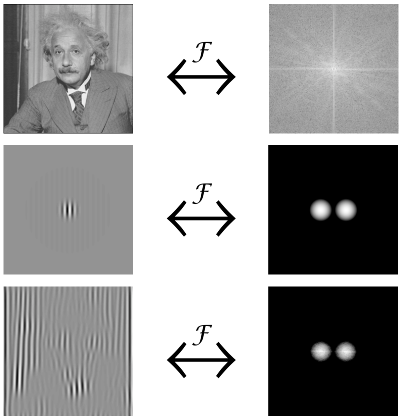


2-D Example: Gabor Filter

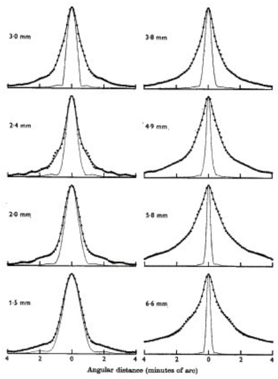
input signal

impulse response

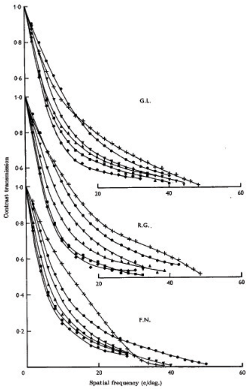
output signal



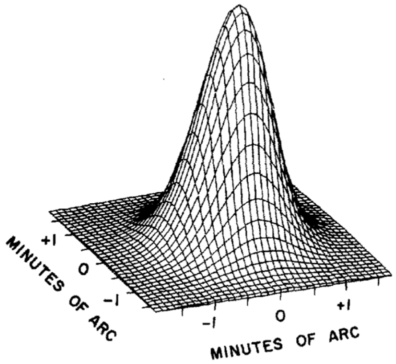
Applications: Line Spread Function (Campbell & Gubisch, 1966)



Applications: Line Spread Function (Campbell & Gubisch, 1966)



Point-Spread Function



Applications: Multiple Mechanisms (Campbell & Robson, 1968)

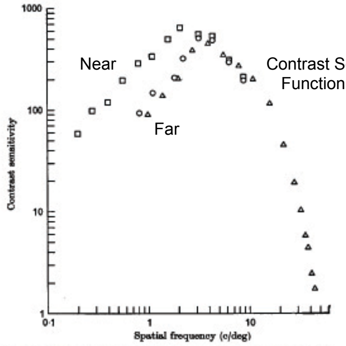


Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture 2° × 2°, △; viewing distance 57 cm, aperture 10° × 10°, □; viewing distance 57 cm, aperture 2° × 2°, ○.

Applications: Multiple Mechanisms (Campbell & Robson, 1968)

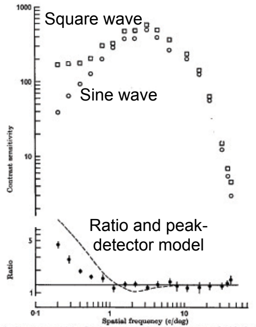


Fig. 3. Contrast sensitivity for sine-wave gratings (○) is compared with that for square-wave gratings (□) for subject J.G.R. at a luminance of 500 cd/m². The ratio of the contrast sensitivities at each spatial frequency is plotted at the bottom of the figure (the bars show ± s.e. of mean). A continuous line is drawn through the ratios at 4/π = 1·273. The dashed line indicates the predicted ratio assuming a simple peak detector mechanism.

Applications: Multiple Mechanisms (Campbell & Robson, 1968)

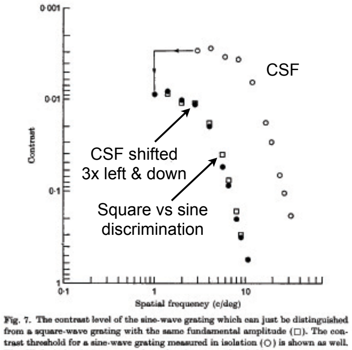
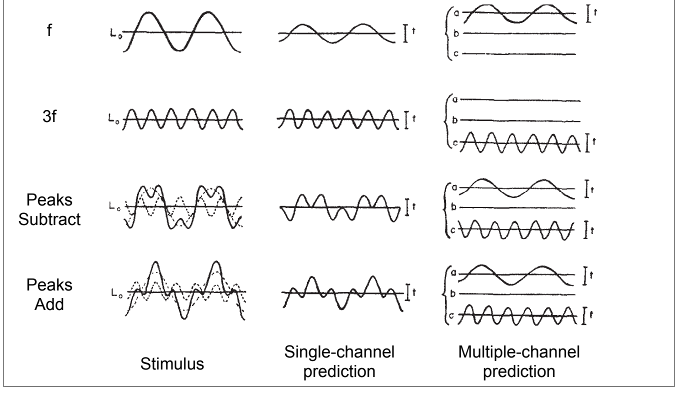
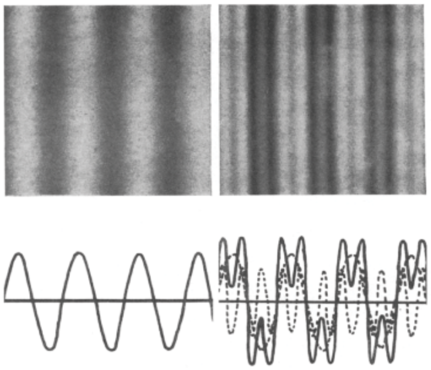


Fig. 7. The contrast level of the sine-wave grating which can just be distinguished from a square-wave grating with the same fundamental amplitude (□). The contrast threshold for a sine-wave grating measured in isolation (○) is shown as well. The filled circles (●) correspond to the sine-wave threshold measurements (○) translated by a factor of 3 in both frequency and contrast as indicated for the lowest frequency point by the arrowed lines. Subject F.W.C., luminance 500 cd/m².

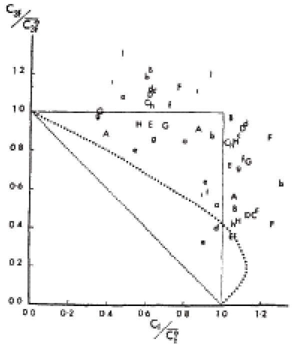
Applications: Summation Within and Between Channels (Graham & Nachmias, 1971)



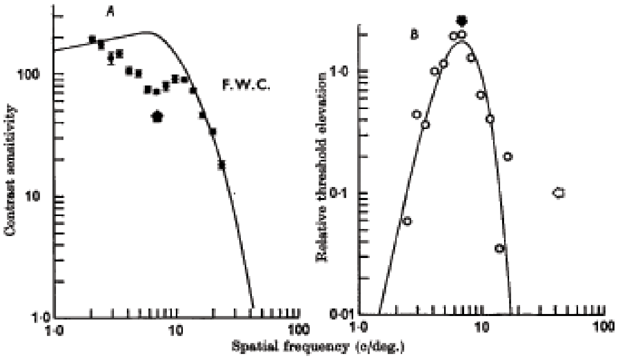
Applications: Summation Within and Between Channels (Graham & Nachmias, 1971)



Applications: Summation Within and Between Channels (Graham & Nachmias, 1971)

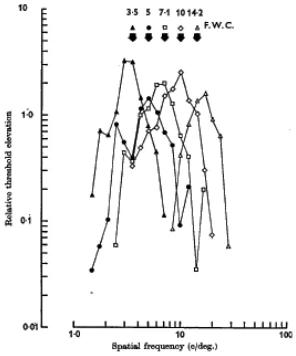


Applications: SF Adaptation (Blakemore & Campbell, 1969)

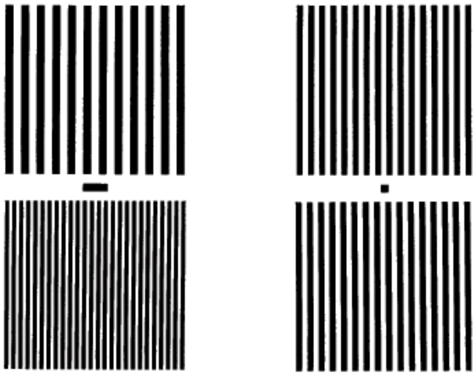


Applications: SF Adaptation (Blakemore & Campbell, 1969)

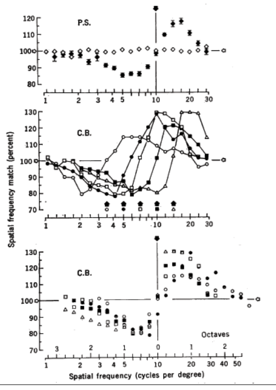
$[e^{-f^2}-e^{-(2f)^2}]^2$ fits all the pooled data.



Applications: SF Adaptation (Blakemore & Sutton, 1969)



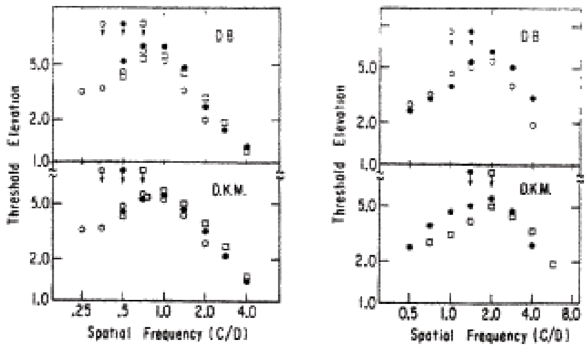
Applications: SF Adaptation (Blakemore & Sutton, 1969)



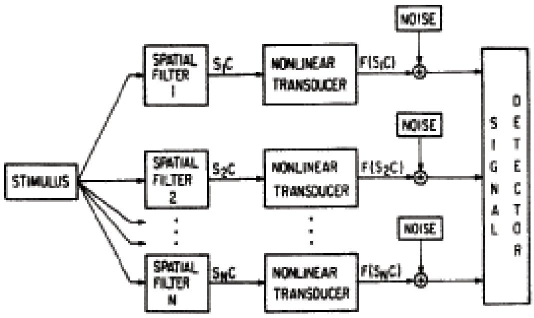
Applications: Pattern Masking (Wilson et al., 1983)

- Mask one pattern (Gabor, D6, ...) by another (e.g., a sine wave grating, tilted obliquely)
- Threshold is raised by the masker if channel being used is sensitive to both
- Many possible explanations of the rise in threshold with masker contrast:
  - Weber's Law, possibly as a result of multiplicative noise (noise whose SD is proportional to mean response): $\dfrac{\Delta I}{I} = k$
  - Nonlinearity followed by additive noise

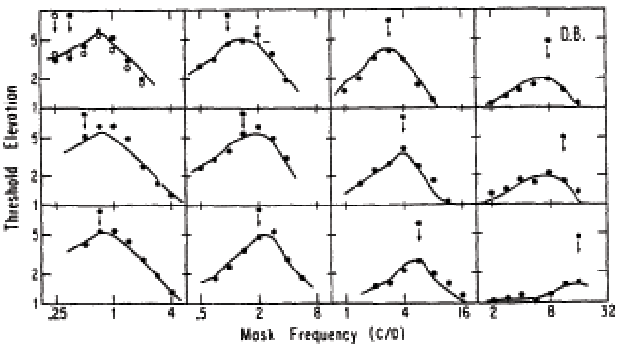
Applications: Pattern Masking (Wilson et al., 1983)



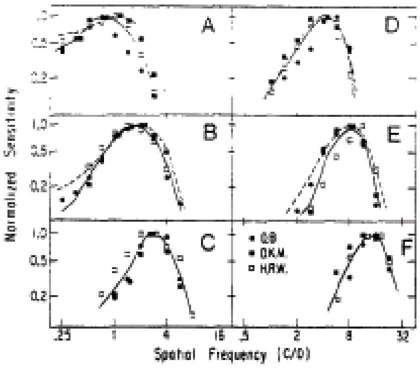
Applications: Pattern Masking (Wilson et al., 1983)



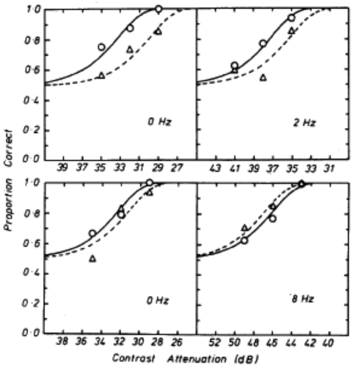
Applications: Pattern Masking (Wilson et al., 1983)



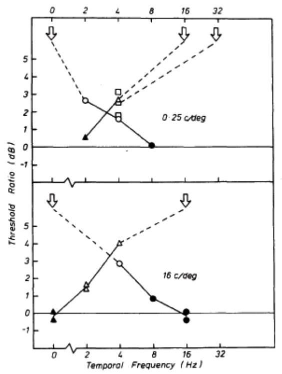
Applications: Pattern Masking (Wilson et al., 1983)



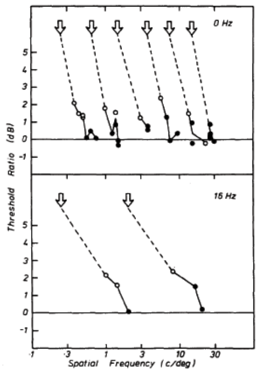
Applications:Detection vs. Identification, Labeled Lines (Watson & Robson, 1981)

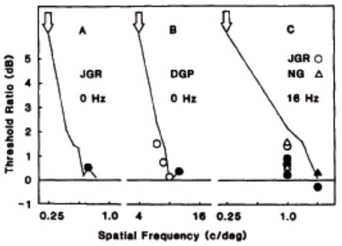


Applications:Detection vs. Identification, Labeled Lines (Watson & Robson, 1981)

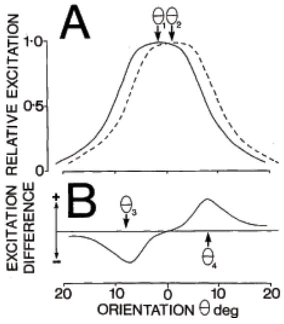


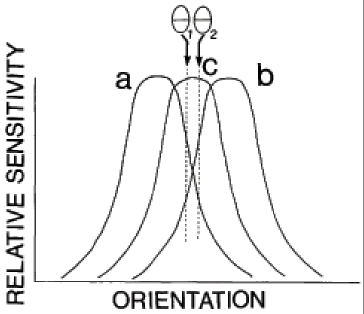
Applications:Detection vs. Identification, Labeled Lines (Watson & Robson, 1981)





Applications:Detection vs. Identification, Most Discriminating Mechanism (Regan & Beverley, 1985)





Applications:Detection vs. Identification, Most Discriminating Mechanism (Regan & Beverley, 1985)

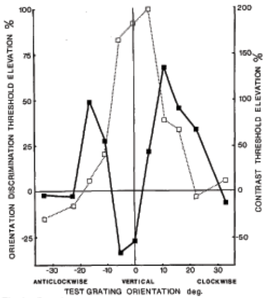
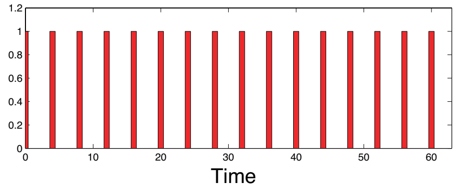
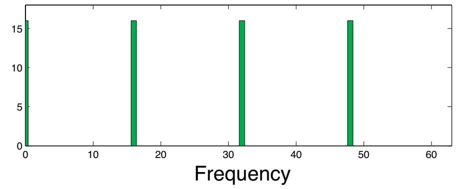


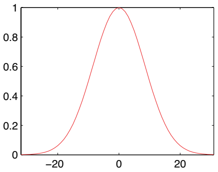
Fig. 3. Postadaptation threshold elevations for orientation discrimination (continuous line) and for contrast detection (dotted line). The adapting grating was vertical (0 on abscissa).

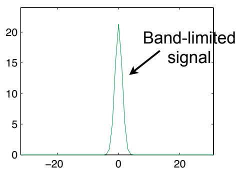
Applications:Sampling and Reconstruction

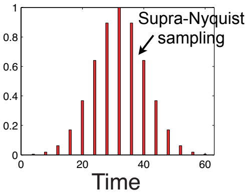


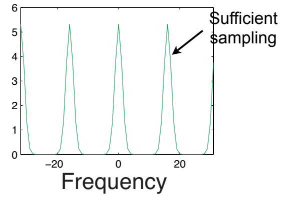


Applications:Sampling and Reconstruction

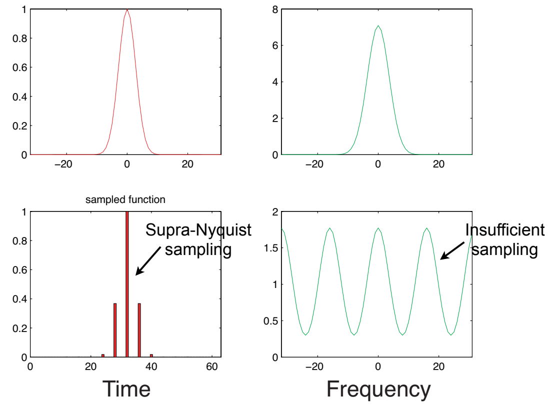




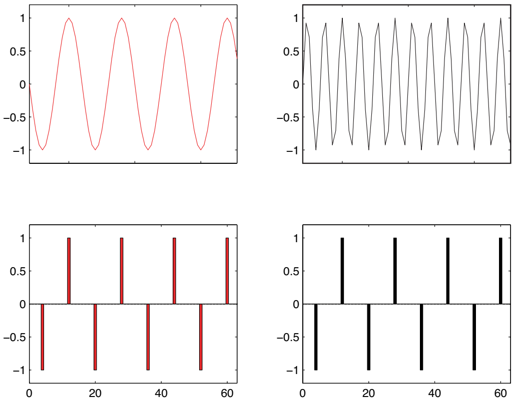




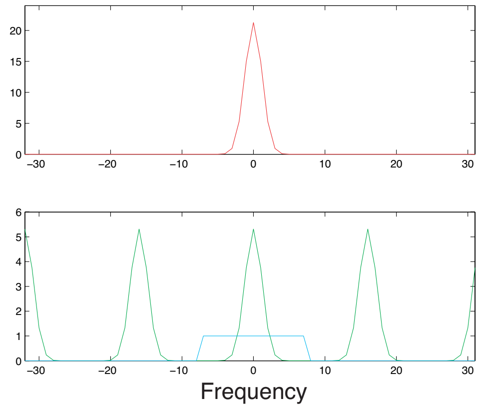
Applications:Sampling and Reconstruction



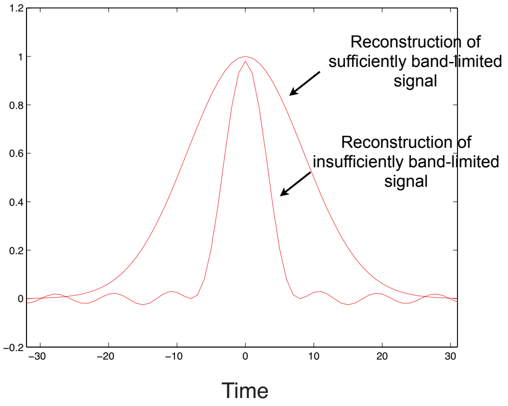
Applications:Sampling and Reconstruction



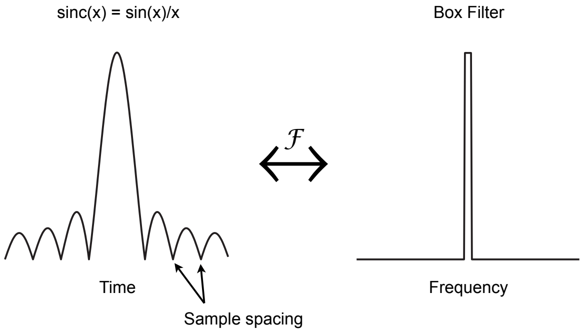
Applications:Sampling and Reconstruction



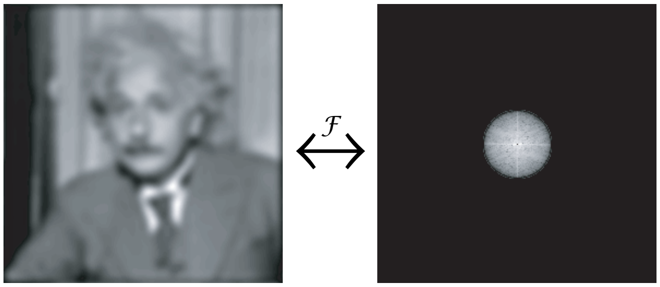
Applications:Sampling and Reconstruction



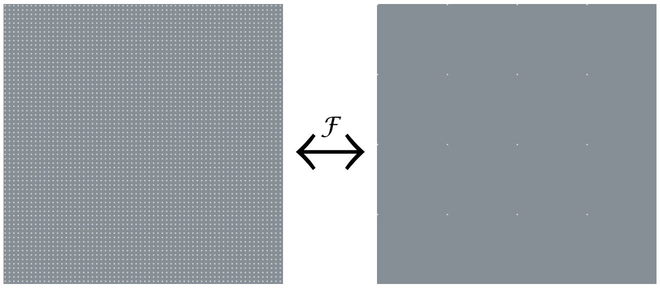
Applications:Sampling and Reconstruction



Applications:Sampling and Reconstruction

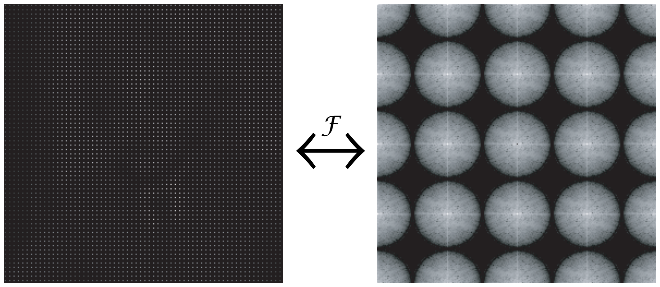


Applications:Sampling and Reconstruction



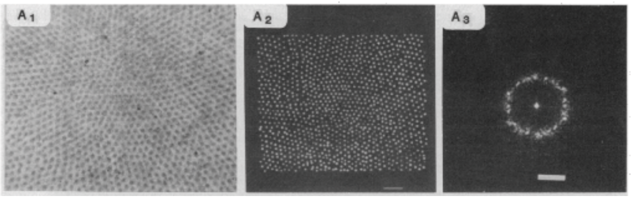
$\overset{\mathcal{F}}{\longleftrightarrow}$

Applications:Sampling and Reconstruction



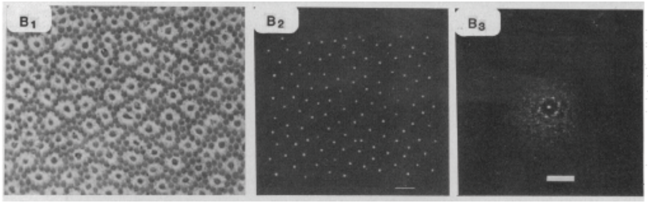
$\overset{\mathcal{F}}{\longleftrightarrow}$

Applications:Sampling and Reconstruction (Yellott, 1983)



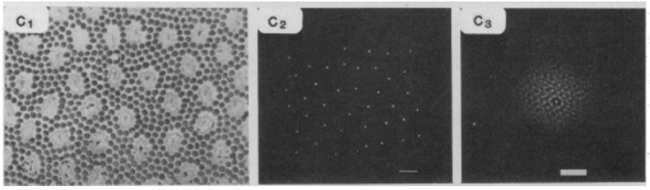
Fovea

Applications:Sampling and Reconstruction (Yellott, 1983)



Parafovea (approx. 6 deg ecc.)

Applications:Sampling and Reconstruction (Yellott, 1983)



Periphery (approx. 35 deg ecc.)