

## Symmetry

“Symmetry” is an important concept in mathematical physics.

When Maxwell initially wove the equations of electricity and magnetism together, he thought they looked unbalanced. He therefore added an equation to make the equations more symmetric. The extra term could be interpreted as creation of a magnetic field by varying an electric field. This turned out to actually exist. Inclusion of the second term allowed trig functions to be solutions to the equations, or electromagnetic waves. Ref?

Around 1900, Henri Poincare and Heinrich Lorentz investigated the math structure of Maxwell’s equations. The celebrated “extra term” turned out to give the EM field a subtle but powerful form of symmetry: a rotation not in space, but in space-time. The effect of rotating in space-time is to project some spatial length into time and vice-versa. It took Einstein’s genius to drive home the full implications. Space and time are not independent, but interwoven. The “rotations” Poincare and Lorentz found in Maxwell’s equations can occur in the real world, through motion. The key to the weird space-time projections lies in the speed of EM waves. Thus there is a deep relationship between EM wave motion and the structure of time and space.

Werner Heisenberg, originator of Quantum Mechanics, argued that what was truly fundamental in nature was not the particles themselves, but the symmetries, or patterns that lay beyond them. These fundamental symmetries could be thought of as the archetypes of matter and the ground of material existence. The particles themselves would simply be the material realizations of those underlying abstract symmetries. These abstract symmetries, normally only ascertainable through mathematics, could be taken as the scientific descendents of Plato’s ideal forms. <sup>1</sup>

### **Paul Davies Superforce**

Chapter 4: Symmetry and Beauty (p. 50-69)

“Perhaps the greatest scientific discovery of all time is that nature is written in mathematical code.” This allows us to understand, predict, and control physical processes.

Notes the relationship between the exponential and sine/cosine functions is an example of a kind of symmetry which physicists look for. (the derivative of  $e^x$  is  $e^x$ ; the fourth derivative of  $\sin$  and  $\cos$  are the  $\sin$  and  $\cos$ ; the deep connection between  $e^x$  and  $\sin x$  is made explicit in the theory of complex numbers. ):

$$E^{ix} = \cos(x) + i \cdot \sin(x)$$

Newton’s laws were completely reformulated by the French physicist Joseph Louis Lagrange and the Irish physicist William Rowan Hamilton in the 19th century.

The conservation laws follow directly from Newton’s laws of motion, but the reformulation of these laws by Lagrange and Hamilton reveal a deep and powerful connection between the

conservation of a quantity and the presence of symmetry. For example, if the system is symmetric when rotated, then it follows from Hamilton's or Lagrange's equations that angular momentum will be conserved.

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Even though the mathematical symmetries may be hard, or even impossible to visualize physically, they can point the way to new principles in nature. Searching for undiscovered symmetries has thus become a major tool of modern physics.

If electric charge is conserved, the question naturally arises as to the nature of the symmetry associated with it. The energy required to lift a weight depends only on the difference in height it is raised; it is independent of route, and of the initial height. "There is a symmetry, therefore, under changes in the choice of zero height. A similar symmetry exists for electric fields. Here voltage is analogous to height. Height and voltage are considered gauge symmetries. It is precisely the gauge symmetry for voltages that assures the conservation of electric charge. Davies SF p 62f

Isotopic spin symmetry: [13] protons are interchangeable with neutrons as long as electric charge is conserved.

The concept of symmetry can be expanded to include things other than time or space. There is a close connection between symmetry and conservation laws. One of the best established conservation laws is that of electric charge. What is the nature of the symmetry associated with conservation of electric charge?

The energy used to lift a weight depends on the change in height, whether measured relative to sea level or ground level; there is a symmetry in changes measured from different heights. A similar symmetry exists for electric fields: voltage plays a role analogous to height. If an electric charge is moved from one point in an electric field to another point, the energy expended depends only on the voltage difference between the end points of its path. If a constant extra voltage is applied to the system, the energy expended does not change. This is an example of another symmetry of Maxwell's equations. These examples illustrate "gauge symmetries": the symmetries involve a "regauging" of height and voltage. These symmetries are not geometrical in nature. It is the gauge symmetry for voltages that ensures the conservation of electric charge.

Symmetry of the strong nuclear force that acts between protons and neutrons. The close similarity of proton and neutrons suggests a symmetry is at work. Nuclear processes would

remain unchanged if we could swap the identity of all protons and neutrons. Strong nuclear force independent of identity transformation from proton to neutron and vice versa; this is called “isotopic spin symmetry.” These symmetry properties are closely analogous to those of intrinsic spin.

Physicists now believe that all forces exist simply to enable nature to maintain a set of abstract symmetries.

What has force got to do with symmetry?

Another definition of symmetric

if it remains unchanged under a certain mathematical operation. The laws of electricity are symmetric under reversal of positive and negative charge. Symmetries used in dealing with the four forces are called gauge symmetries. Gauge symmetries have to do with re-gauging the level or scale or value of some physical quantity, and a system possesses gauge symmetry if the physical nature of the system remains unchanged under such an alteration. [22] p.112

Here is a deep principle of nature: physics in a curved path around a planet (in a spacecraft, for instance) is the same as in a straight path in deep space. The reason is that the gravity of the planet exactly neutralizes the effect of curving of the path of the spacecraft. The conclusion:

The laws of physics can be made symmetric even under local gauge transformations in distance provided a gravitational field is introduced to compensate for the place to place variations. Or, the gravitational field is nature's way of maintaining a local gauge symmetry. A freedom to regauge the scale of distance arbitrarily from place to place. With gravity, we can change to any shape of path whatever without changing the physics. The symmetry here is the invariance of physics under arbitrary changes in the shape of the path of motion. Viewed this way, gravity is simply a manifestation of an abstract symmetry—a local gauge symmetry. Not just gravity, but all four but all four forces can be treated in this way: they can all be regarded as gauge fields. In a quantum description of gauge fields coupled to particles of matter the concept of a gauge change must be widened further and related to the phase of the quantum wave which describes a particle. Nature exhibits a number of local gauge symmetries and is compelled to introduce several force fields to compensate for the gauge changes involved. The EM field is a manifestation of the simplest known gauge symmetry that is consistent with the principles of special relativity. The gauge transformations involve changes in voltage from place to place.

It is intriguing that the existence of electromagnetism could be deduced from two requirements:

The simplest local gauge symmetry and the so-called Lorentz-Poincare symmetry of special relativity. [23] p.114

Gauge symmetry is the key to constructing quantum theories of the forces that are free of the destructive infinite terms discussed previously. [24] p. 115. Note this in discussion of ZPF

**Wikipedia:** [http://en.wikipedia.org/wiki/Introduction\\_to\\_gauge\\_theory](http://en.wikipedia.org/wiki/Introduction_to_gauge_theory)

Modern physical theories describe reality in terms of fields, e.g., the electromagnetic field, the gravitational field, and fields for the electron and all other elementary particles. A general feature of these theories is that none of these fundamental fields, which are the fields that change under a gauge transformation, can be directly measured. On the other hand, the observable quantities, namely the ones that can be measured experimentally — charges, energies, velocities, etc. — do not change under a gauge transformation, even though they are derived from the fields that do change. This (and any) kind of invariance under a transformation is called a symmetry.

For example, in classical electromagnetism the electric field,  $E$ , and the magnetic field,  $B$ , are observable, while the underlying and more fundamental electromagnetic potentials  $V$  and  $A$  are not.[3] Under a gauge transformation which jointly alters the two potentials, no change occurs either in  $E$  or  $B$  or in the motion of charged particles. In this example, the gauge transformation was just a mathematical feature without any physical significance, except that gauge invariance is intrinsically connected to the fundamental law of charge conservation

Historically, the first example of gauge symmetry to be discovered was classical electromagnetism. A static electric field can be described in terms of an electric potential (voltage) that is defined at every point in space, and in practical work it is conventional to take the Earth as a physical reference that defines the zero level of the potential, or ground. But only differences in potential are physically measurable, which is the reason that a voltmeter must have two probes, and can only report the voltage difference between them. Thus one could choose to define all voltage differences relative to some other standard, rather than the Earth, resulting in the addition of a constant offset.[4] If the potential  $V$  is a solution to Maxwell's equations then, after this gauge transformation, the new potential

$V > V+C$  is also a solution to Maxwell's equations and no experiment can distinguish between these two solutions. In other words the laws of physics governing electricity and magnetism (that is, Maxwell equations) are invariant under gauge transformation.[5] That is, Maxwell's equations have a gauge symmetry.

Generalizing from static electricity to electromagnetism, we have a second potential, the magnetic vector potential  $A$ , which can also undergo gauge transformations. These transformations may also be local. That is, rather than adding a constant onto  $V$ , one can add a function that takes on different values at different points in space and time. If  $A$  is also changed in certain corresponding ways, then the same  $E$  and  $B$  fields result. The detailed mathematical relationship between the fields  $E$  and  $B$  and the potentials  $V$  and  $A$  is given in the article Gauge fixing, along with the precise statement of the nature of the gauge transformation. The relevant point here is that the fields remain the same under the gauge transformation, and therefore Maxwell's equations are still satisfied.

[http://en.wikipedia.org/wiki/SU\(2\)](http://en.wikipedia.org/wiki/SU(2))

The special unitary group of degree  $n$ , denoted  $SU(n)$ , is the group of  $n \times n$  unitary matrices with determinant 1. The group operation is that of matrix multiplication. The special unitary group is a subgroup of the unitary group  $U(n)$ , consisting of all  $n \times n$  unitary matrices,

The  $SU(n)$  groups find wide application in the Standard Model of particle physics, especially  $SU(2)$  in the electroweak interaction and  $SU(3)$  in QCD

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<sup>1</sup> F. David Peat *Synchronicity: The Bridge Between Matter and Mind* p. 94 f.