Kurt Godel

Gödel's theorem is a result about axiomatic systems. In ordinary educated speech, axioms are undoubtable truths.

An axiomatic system consists of some undefined terms, a bunch of axioms referring to those terms and partially describing their properties, and a rule or rules for deriving new propositions from already existing propositions.

An axiomatic system is said to be *consistent* if, given the axioms and the derivation rules, we can never derive two contradictory propositions;

There are two very common but fallacious conclusions people make from this, and an immense number of uncommon but equally fallacious errors I shan't bother with. The first is that Gödel's theorem imposes some of profound limitation on knowledge, science, mathematics. Now, as to science, this ignores in the first place that Gödel's theorem applies to *deduction from axioms*, a useful and important sort of reasoning, but one so far from being our only source of knowledge it's not even funny. It's not even a very common mode of reasoning in the sciences, though there are axiomatic formulations of some parts of physics. Even within this comparatively small circle, we have at most established that there are some propositions about numbers which we can't prove formally. As Hintikka says, "Gödel's incompleteness result does not touch directly on the most important sense of completeness and incompleteness, namely, descriptive completeness and incompleteness," the sense in which an axiom systems describes a given field. In particular, the result "casts absolutely no shadow on the notion of truth. All that it says is that the whole set of arithmetical truths cannot be listed, one by one, by a Turing machine." Equivalently, there is no algorithm which can decide the truth of *all* arithmetical propositions.
http://cscs.umich.edu/~crshalizi/notabene/godels-theorem.html
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Gödel's contribution to the study of logic revolves around the discovery of a limitation on what one can sensibly seek from a logical system. Prior to his work, a great many mathematicians, vaguely headed by the likes of Bertrand Russel, were hard at work trying to prove consistency and completeness of the Zermelo-Fränkel formalism for set theory and logic, together with Peano's axioms for the natural numbers. In so far as they were able to believe that this might fail, they expected the failure to show them how to replace their formalism with one which would be consistent and complete. Gödel derailed this project in the most spectacular manner possible: he showed that any logical system capable of supporting Peano's axioms (a formal description of the process of counting) could not be both consistent and complete.
http://www.chaos.org.uk/~eddy/math/Godel.html

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Jones and Wilson, *An Incomplete Education*

all logical system of any complexity are, by definition, incomplete; each of them contains, at any given time, more true statements than it can possibly prove according to its own defining set of rules.
Gödel's Theorem … has been taken to imply that you'll never entirely understand yourself, since your mind, like any other closed system, can only be sure of what it knows about itself by relying on what it knows about itself.

Boyer, *History of Mathematics*

Gödel showed that within a rigidly logical system such as Russell and Whitehead had developed for arithmetic, propositions can be formulated that are undecidable or undemonstrable within the axioms of the system. … It appears to foredoom hope of mathematical certitude through use of the obvious methods. Perhaps doomed also, as a result, is the ideal of science - to devise a set of axioms from which all phenomena of the external world can be deduced.

Nagel and Newman, *Gödel's Proof*

Second main conclusion is ... Gödel showed that *Principia*, or any other system within which arithmetic can be developed, is *essentially incomplete*. In other words, given *any* consistent set of arithmetical axioms, there are true mathematical statements that cannot be derived from the set... Even if the axioms of arithmetic are augmented by an indefinite number of other true ones, there will always be further mathematical truths that are not formally derivable from the augmented set.

Rucker, *Infinity and the Mind*

…Although this theorem can be stated and proved in a rigorously mathematical way, what it seems to say is that *rational thought can never penetrate to the final ultimate truth* ...

Hofstadter, *Gödel, Escher, Bach*

… how can you tell if your own logic is "peculiar' or not, given that you have only your own logic to judge itself?

…The other metaphorical analogue to Gödel's Theorem which I find provocative suggests that ultimately, we cannot understand our own mind/brains ... Just as we cannot see our faces with our own eyes, is it not inconceivable to expect that we cannot mirror our complete mental structures in the symbols which carry them out? [but we can see our own eyes with mirrors]

http://www.miskatonic.org/godel.html

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