dispersion relations

http://en.wikipedia.org/wiki/Dispersion_%28optics%29 dispersion in optics; Group Dispersion Velocity; negative group velocity speed of light; group velocity speed of light exceeds C

http://en.wikipedia.org/wiki/Dispersion_relation

<http://www.brainpop.com/math/>

google electromagnetic waves in plasma

dispersive waves: em wave propagated thru plasma and conductors

We, thus, conclude that the spatial extent of the pulse grows *linearly* in time, at a rate proportional to the second derivative of the dispersion relation with respect to k (evaluated at the pulse's central wavenumber). This effect is known as *pulse dispersion*. In summary, a wave pulse made up of a linear superposition of dispersive sinusoidal waves, with a range of different wavenumbers, propagates at the *group velocity*, and also *gradually disperses* as time progresses.

<http://farside.ph.utexas.edu/teaching/315/Waves/node47.html>

We, thus, conclude that sinusoidal electromagnetic waves propagating through a plasma have a *nonlinear* dispersion relation. Moreover, it is clear that this nonlinearity arises because the effective refractive index of the plasma is *frequency dependent*.

The expression [\(595\)](http://farside.ph.utexas.edu/teaching/315/Waves/node48.html#e9.29) for the refractive index of a plasma has some rather unusual $\omega > \omega_p$

properties. For wave frequencies lying above the plasma frequency (*i.e.*, $\qquad \qquad$), it yields a real refractive index which is *less than unity*. On the other hand, for wave

 $\omega < \omega_p$ frequencies lying below the plasma frequency (*i.e.*, $\qquad \qquad$), it yields an *imaginary* refractive index. Neither of these results makes much sense. The former result is problematic because if the refractive index is less than unity then the wave *phase*

 $v_p = \omega/k = c/n$ $v_p > c$ *velocity*, becomes *superluminal* (*i.e.*, \sim), and superluminal velocities are generally thought to be unphysical. The latter result is problematic because an imaginary refractive index implies an imaginary phase velocity, which seems utterly meaningless. Let us investigate further.

 $\omega > \omega_{\rm p}$

According to [\(595\)](http://farside.ph.utexas.edu/teaching/315/Waves/node48.html#e9.29), a sinusoidal electromagnetic wave of angular frequency propagates through the plasma at the superluminal phase velocity

$$
v_p = \frac{\omega}{k} = \frac{c}{n} = \frac{c}{(1 - \omega_p^2/\omega^2)^{1/2}}.
$$
\n(596)

But, is this really unphysical? As is well-known, Einstein's theory of relativity forbids *information* from traveling faster than the velocity of light in vacuum, since this would violate *causality* (*i.e.*, it would be possible to transform to a valid frame of reference in which an effect occurs prior to its cause.) However, a sinusoidal wave with a unique frequency, and an infinite spatial extent, does not transmit any information. (Recall, for instance, from Section 8.3 , that the carrier wave in an AM radio signal transmits no information.)

The above analysis demonstrates that a sinusoidal electromagnetic wave cannot propagate through a plasma when its frequency lies below the plasma frequency. Instead, the amplitude of the wave *decays exponentially* into the plasma. Moreover, the $\pi/2$

electric and magnetic components of the wave oscillate in *phase quadrature* (*i.e.*, radians out of phase), and the wave consequentially has *zero* associated net energy flux. This suggests that a plasma *reflects*, rather than absorbs, an incident electromagnetic wave whose frequency is less than the plasma frequency

The outer regions of the Earth's atmosphere consist of a tenuous gas which is *partially ionized* by ultraviolet and X-ray radiation from the Sun, as well as by cosmic rays incident from outer space. This region, which is known as the *ionosphere*, acts like a plasma as far as its interaction with radio waves is concerned.

<http://farside.ph.utexas.edu/teaching/315/Waves/node48.html>